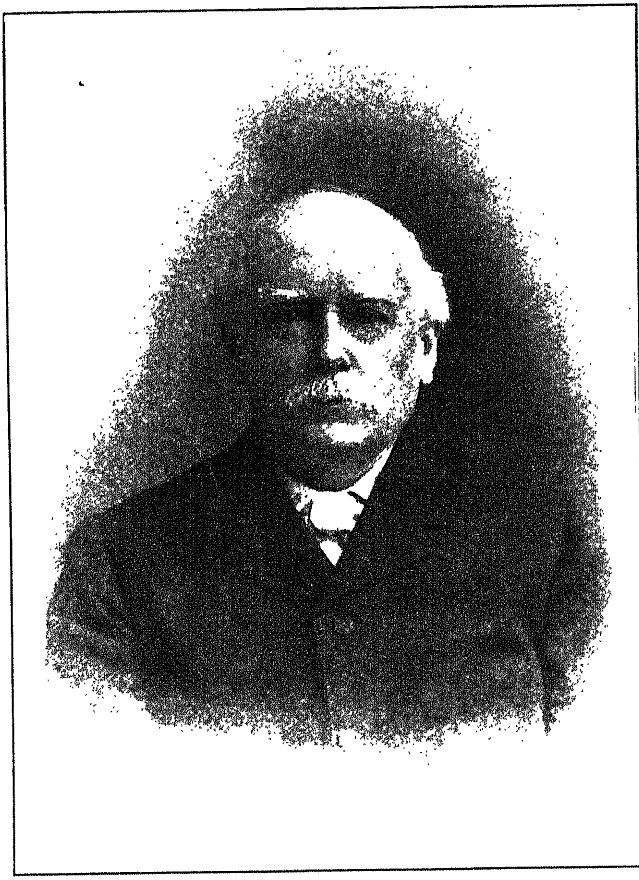
DYNAMOMETERS



THE AUTHOR.

DYNAMOMETERS

BY

REV. FREDERICK JOHN JERVIS-SMITH, M.A., F.R.S.

EDITED AND AMPLIFIED BY

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EDITOR'S PREFACE

In completing and preparing for the press the unfinished manuscript of the author, I have had the advantage from long and intimate association with him of knowing his views on the subject generally. Unfortunately I had not had any discussion with him upon the scheme of his book, and where the parts succeed one another in a manner which is somewhat disjointed I do not know whether he intended to introduce paragraphs which would connect these parts.

From his table of contents I have learned without doubt the order he intended to follow, and to this I have adhered faithfully. I have occasionally made minor corrections or explanations where I thought them desirable, and these are not indicated. Additional paragraphs are indicated by square brackets [ ]. For any faults in these the author was in no way responsible. I have not thought it necessary or desirable to abstract every printed account of a dynamometer which I have found, but I have described in sufficient detail some for which he had left blank spaces, and a number of others which seemed to me to be of sufficient interest. My regret is that I may have overlooked so many that should have been included.

It has been a pleasure to me to do my best to complete this book, for it shows how the author from the time that he was acting as his father's curate at Taunton, and while he was Millard Lecturer on Mechanics at Oxford, devoted himself consistently to the science and art of dynamometry, and in that time devised and constructed nearly every known type of instrument, in many cases being the pioneer.

C. V. B.

OBITUARY NOTICE

(From the Proceedings of the Royal Society.)

FREDERICK JOHN JERVIS-SMITH, 1848—1911.

The Reverend Frederick John Jervis-Smith, the only son of the Reverend Prebendary Smith, of Taunton, was born at Taunton on April 2, 1848. He was educated at Pembroke College, Oxford. While still a boy at home he had the great advantage of meeting constantly William Ellis Metford, by whom his natural aptitude for science and mechanics was stimulated so that his genius, which was so marked in these directions, forced him at a later date to break away from the narrower life which his father wished him to follow. In obedience to this wish he entered the Church and acted for some years as his father's curate and organist, becoming later Vicar and Patron of the living of St. John's, Taunton. It must have been at this time also that with the help of Sir John Stainer he attained that knowledge of music and skill at the organ and piano that his friends so greatly admired, for a touch such as his could not have been acquired in later life.

While at Taunton he followed the bent that was so strong in him and carried on experimental work in his own workshop, acquiring by various means an intimate knowledge of workshop practice such as the amateur rarely possesses. In 1886 he was invited to take charge of the Millard Engineering Laboratory attached to Trinity College, Oxford, and it was here that his best work was done.

A good indication of the variety of Jervis-Smith's investigations may be found by reference to the *Philosophical Magazine* in the twenty years from 1882 to 1902. The subject to which

he devoted himself most particularly was that of work-measuring machines and integrators, and many of the papers are on this subject. Several papers refer to the measurement of the torsion of rotating shafts with a view to determine the power being transmitted, and one of his early papers describes the means now adopted on large steamships, where, owing to the engines being turbines, indicated power cannot be ascertained and the torsional method is the only one available.

Other enquiries which interested him were the magnetic properties of metals as affected by mechanical stress or by heat; electric sparks and the influence on them of flame or pressure. Under this heading, probably, should be mentioned his beautiful electrically produced images of coins that he called inductoscript.

One of the most valuable results of Jervis-Smith's ingenuity and mechanical aptitude is his tram chronograph. Those who have used the old pendulum myographs so usual in physiological laboratories, where the time records are rendered tiresome by the variable speed of the recording surface, should be the first to appreciate this beautiful instrument, in which trouble from this cause is entirely eliminated. A still greater value has been given to this instrument by the perfection of the electromagnetic styles that he invented and made. By making his electromagnets extremely small and the yoke relatively short and thick he reduced the latent period, so that this chronograph is now not only the most convenient but the most accurate instrument for ballistic and other measurements of the kind.

Other subjects of less interest perhaps in which Jervis-Smith made investigations or inventions were in relation to mercury pumps and means for raising the mercury continuously and automatically, quick distillation of mercury *in vacuo*, recalescence of iron, and high resistances made of graphite and plaster of Paris.

During the last few years since his retirement to his charming house near Lymington Jervis-Smith was greatly interested in glowing phenomena in vacuous bulbs moved or spun in electric

and magnetic fields. On these he made numerous original experiments, but up to the present these results are not well understood.

Jervis-Smith was awarded a medal at the Paris Exhibition of 1878 for a dynamometer, and at the Inventions Exhibition at South Kensington he was awarded a silver medal for his work on dynamometers. He also received a medal from the Royal Humane Society for rescuing a person in danger of being drowned. He was a member of the Committee on Explosives appointed by the Home Office in 1895—96. He became a Fellow of the Royal Society in 1894.

He was keenly interested in the historical side of Physical Science, and often brought to light curious anticipations of more recent inventions. He found, for instance, that the telephone had been made and described in Italy as an instrument for recording taps upon it by movement at the receiving end. The former inventor had apparently invented the same instrument as Bell, but he never thought of speaking into it. This historical appreciation made the selection of Jervis-Smith to represent the University of Oxford at the tercentenary of Torricelli at Faenza in 1898 singularly appropriate.

Throughout his career one subject was constantly receiving his attention, and that was dynamometry in its widest sense. On this he had been collecting papers all his life, and in his later years he was putting these in order in the hope of seeing the great work completed which had gained so much from his originality. It is hoped that this will appear this year.

He married Annie Eyton, second daughter of T. Taylor, Esq., who with one surviving son remains to mourn his loss.

The singular charm, humour, and modesty of Jervis-Smith, no less than his genius, made his friendship a valued possession. The writer of this notice found in addition a community of taste and a mutual sympathy, and he has lost his closest and most valued friend and counsellor.

C. V. B.

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CHAPTER I

INTRODUCTION

In writing this book it has been my aim to place before the reader an account of some of those machines which have from time to time been invented with a view to estimate the output of prime-movers, and the power absorbed by machines when driven by engines or motors. The subject-matter is therefore mainly historical. It is not my intention to describe fully the dynamometric methods employed in measuring ship resistances, by means of ship models drawn through water. Of late several testing tanks have been built, in England, France, Germany, and Japan, modelled on the original Testing Tank

of William Froude. The subject of ship model testing ha become so large and varied that it would be quite beyond th scope of this book to give a full and adequate account of thi branch of Dynamometry. In Chapter XV. a list of ship testing plants will be found, including some of the construc tional details. As the powers employed by engineers hav increased during no long period of time from a few to 10,00 horse-power, machines for measuring such powers have bee modified, and designed to give accurate results under th different requirements of the cases to be dealt with. It i interesting to observe that in one of the earliest dynamo meters, namely that of Hirn, the method employed o measuring power is the same as that used in the latest form o steamship propeller dynamometer, the horse-power delivere being, in both cases, found by measuring the torsion of shaft. In the description of different dynamometers I hav added figures and diagrams, in order to make the mechanica constructions clear when necessary. Since Prony, Hirn Morin, Thomson (Lord Kelvin), and Froude may be calle the founders of dynamometric measurement, I have give descriptions of the machines of the last four from their ow original papers, practically in full.

The inventions of these pioneers of the subject are remark able. Prony was the first to employ the friction set up betwee two surfaces, as a measured resistance to a pulley whee rotated by a prime-mover.

Hirn made his torsional dynamometer totalise the worl done during a given period of time. Morin added an integrato to his transmission dynamometer, and also an automati recording apparatus, which exhibited the work done as a area, generated on a moving paper band; by this means no only was the total *work* shown, but also the way in which i was built up during any period of time. Thomson devised brake machine in which the moment of inertia of the brake namely a rope, was reduced to the least workable value thereby minimising the tendency to oscillate in the brak itself, and rendering the dynamometer steady when running To Froude is due the turbine brake, in which enormou resistance to rotation is obtained in a small space by the usefu application of the vortical rotation of a liquid.

Some years ago I published a pamphlet in which a short account was given of certain forms of work-measuring machines. It was little more than a sketch of some experiments which were made in Paris in 1881, and at Taunton in a private laboratory, on dynamometric measurements. Since then more experiments on the same subject have been carried on by me and some of the students at the Millard Engineering Laboratory at Oxford (1886 to 1903). Believing that some portions of the work may be of interest to engineers, science students and others not directly engaged in the design of machines and engines, I have collected in the following pages some further accounts of dynamometric experiments and calculations bearing on the subject, so that the reader may be able to compare the different methods which have been employed by investigators of eminence in estimating the output and efficiency of prime-movers and mechanical combinations of many different kinds. Now that a certain class of machinery, including the dynamo, the electric motor, and many kinds of engines, employed in ships and launches, railways, trams, motor cars, and flying machines, has undergone rapid development, and in some cases reached a high degree of perfection, it has become necessary that some adequate method of testing the comparative merits of such machines should be in the hands of those who make them and use them.

One of the leading features of excellence of machinery, either of prime-movers or machines driven by them, is economy of working, so that accurate dynamometric tests of such machines is of paramount importance.

An observation made by John Penn, the celebrated marine engineer, chairman of the Institution of Mechanical Engineers, in 1858, at the close of a paper by William Froude on dynamometers, is interesting in connection with my subject. It is as follows :—" He " (Mr. Penn) " thought there could not be too many dynamometers, as they were of such importance." Penn's words were spoken at a time when the available methods for measuring efficiency were quite few and not much known ; he evidently felt that progress in the construction of the steam engine could be best gauged by dynamometric measurements of work. The masterly experiments of the late W. Froude, F.R.S., on the power absorbed by marine engines, and ship

resistance, decided certain dynamometrical questions so perfectly that the importance of this class of measurement was thrust with great force upon naval architects and the builders of marine engines. Dynamometric tests bearing on the structure of our battleships are now carried on daily at the Admiralty experimental works at Haslar under the direction of Mr. E. Froude, F.R.S., and similar testing plants have been laid down by shipbuilders who wish to conduct their own testing. [These have been added to recently by the construction of a tank at the National Physical Laboratory at Bushy.] Some few makers of electrical machines long ago (1881) realised the importance of dynamometric tests; also excellent tests of this kind were made from time to time under the supervision of the Royal Agricultural Society of England. In 1888-89 the Society of Arts conducted exhaustive trials of motors for electric lighting. At Messrs. Willans and Robinson's Engineering Works, Rugby, a complete department for engine testing has been arranged by Captain Riall Sankey and Mr. C. H. Wingfield. In a well-illustrated paper by Mr. W. W. Beaumont,* will be found detailed descriptions of dynamometers of the friction-brake type. My own observations from having visited many engineering works and technical colleges, both in England and in the colonies of Australia and New Zealand, lead me to believe that much has yet to be done in this most interesting and necessary branch of experimental work. The dynamometric test either of a prime-mover such as a steam or gas engine, or of a driven machine such as a dynamo or spinning loom, should form a definite part of the workshop procedure of the mechanical and the electrical engineer, since such tests would definitely show progress or the reverse in the machine produced in the works.

It has been customary with engineers to call the machine used for measuring work a dynamometer. The name does not seem to have been well chosen, since the word, derived as it is from the Greek δύναμὶς = force, and μετρον = a measure, would imply that only force was measured, whereas work, *i.e.*, the product of force into the space through which it acts, is measured by the machine. Since this is the case, a word

* Proceedings of the Institution of Civil Engineers, Vol. XCV., November, 1888.

compounded of the Greek words εργον = a work, and μετρον = a measure, would be preferable. Machines whereby work is measured have been called ergometers (see "Elements of Natural Philosophy," Thompson and Tait, p. 134, Part I., 2nd ed., 1879). But as the word "dynamometer" has been so universally employed and applied to work-measuring machines it will be adhered to in this book. Dynamometers may be divided into three classes, the gravity, the frictional (including absorption machines), and the transmission forms. These and some of the methods used for obtaining numerical results will be considered in order.

If we wished to measure the work done by any engine, whether driven by water, steam, gas, electricity, or any other agent, we could make such a measurement by causing the engine or motor to wind up a known weight from a deep shaft in the earth; then the height in feet through which the weight is raised, at a uniform velocity, multiplied by the numerical value of the total weight in pounds raised would give the numerical value of work done in raising the load in *foot-pounds*. In order that this very simple method might be practically employed the winding-rope should be continuous, so that the rope in itself may be in equilibrium. If an unbalanced rope were employed, its weight must be known; then the product of the height through which its centre of gravity is raised multiplied by its weight = the work done in raising the rope. The sum of the two products equals the whole work done.

Probably the earliest instance of carefully-conducted experiments on the determination of the work done by a machine in a certain time is due to John Smeaton, F.R.S. I give two instances of his methods of measuring *work*.

The vanes of a small windmill were carried on an axle mounted on an arm FG (Fig. 1) perpendicular to its length, the arm being capable of rotation in a horizontal plane, on the axle DE, mounted on the frame ABC. By means of a cord ZH coiled on the vertical axle, the little windmill was driven through the air at any required velocity; while the windmill was thus impelled against the air it raised known weights P placed in a pan ST to a known height by means of pulleys and a cord MNO. Smeaton's experiments were made on windmills furnished with vanes varying in shape and angular set.

A comparison between the values of the weights raised in different cases indicated the relative values of the different

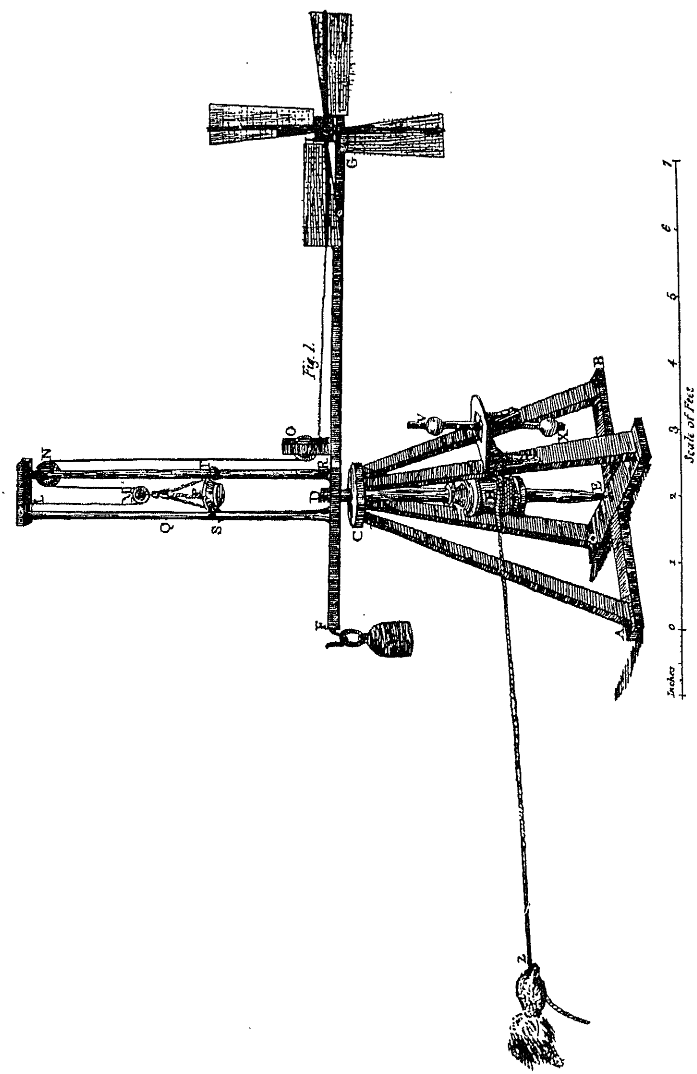

FIG. 1.

forms of vanes and their angular set. A pendulum VX was used to show the time in which *work* was done. In another

set of experiments Smeaton employed a somewhat similar method to determine the work done in generating angular velocity in a certain given mass, starting from rest when impelled by a force acting through a given space, whereby he essayed to find the relationship existing between rotating bodies and a falling weight which propelled them. Here we see probably for the first time *work* expressed as a product. Smeaton writes thus: "And this mechanic power we shall express by the number 202, the product of ounces in the scale multiplied by the inches in its perpendicular descent, for $8 \times 25\frac{1}{2} = 202$"—about 1·06 foot-pounds. Smeaton's unit of *work* was the inch-ounce, a product like the foot-pound now used by the modern engineer. It will be noted that Smeaton, by means of his pendulum, determined the time also in which *work* was done; so that the *rate* of doing *work*, that is *power*, was found by him. In these days of aeroplane flights the earliest definite research in windmill sails should be of interest to those who are working on the behaviour of air screw propellers. It will be seen in what follows that Smeaton's excellent method has recently been developed into elaborate machines for testing the power of aerial propellers (Chap. XVI.).

Some excellent experiments were made by M. Le Chevalier De Borda on the horizontal impulse wheel by means of the gravity method ("Mémoire sur les Roues Hydraulique," par M. le Chevalier De Borda, Hist. de l'Acad. Royal des Sciences, 1767). The water struck the vanes or curved blades of the wheel at an angle to the vertical; De Borda caused the hydraulic wheel to raise a weight by means of a pulley and a cord wrapped on the axle of the wheel, and by this means determined the condition of the maximum efficiency.

Atwood, the inventor of the well-known machine which bears his name, when writing on the dynamics of rotating bodies (1784), gives an illustration of Smeaton's instrument, but in his calculations the kinetic energy of the driving mass is included in the reckoning, and also other important points ("A Treatise on the Rectilinear Motion and Rotation of Bodies," G. Atwood, M.A., F.R.S., Cambridge, 1784). An experiment involving the same dynamic condition is described in "Applied Mechanics," by Prof. John Perry, F.R.S., ed. of 1897, p. 247.

Soon after the steam engine of James Watt had shown itself to be an excellent prime-mover and the horse began to be replaced by it, the need of some method for determining the comparative power of the steam engine and the horse was felt.

The following quotation from John Farey shows clearly how the early estimates of horse-power were arrived at and determined. The passage is important historically, and therefore I give it at full length.

Estimation of the Force of Steam Engines by Horse-power, 1784 ("The Steam Engine," by John Farey: London, 1827):—

"The only unequivocal mode of expressing the mechanical power exerted by an engine or by an animal is the weight which can be raised through a certain space in a given time by that exertion; and unless we define what a horse-power is in those terms, it is a very vague expression, on account of various degrees of strength which different horses possess, and their capacity of enduring fatigue."

"When Messrs. Boulton and Watt first began to introduce their rotative steam engines into manufactories, about 1784, they found it necessary to adopt some measure of the power which they were required to exert; this they endeavoured to do in such terms as would be readily understood by the persons who were likely to want such engines. The machinery in the great breweries and distilleries in London was then moved by the strength of horses, and the proprietors of those establishments, who were the first to require Mr. Watt's engines, always inquired what number of horses an intended engine would be equal to.

"In consequence, Mr. Watt made some experiments on the strong horses employed by the brewers in London, and found that a horse of that kind, walking at the rate of two and a half miles per hour, could draw 150 lb. avoirdupois, by means of a rope passing over a pulley, so as to raise up that weight, with vertical motion, at the rate of 220 ft. per minute.

"This exertion of mechanical power is equal to 33,000 pounds (or 528 cubic ft. of water) raised vertically through a space of one foot in one minute, and he denominated it a horse-power, to serve for a measure of the power exerted by his steam-engines; that is, of the resistance actually overcome, in addition to the friction of the engine itself, and the resistance of the air pump. . . . Messrs. Boulton and Watt's standard for the horse-power is very much beyond the actual power of any horse, except the very strongest, and they cannot long endure the exertion of raising 33,000 lb. at the rate of 1 ft. per minute. Mr. Smeaton and other

engineers made many observations on the work actually performed by horses when working regularly in mills, and the results seem to show that 22,000 lb., raised at the rate of 1 ft. per minute, may be taken for a real horse-power, or as the exertion that a good horse can overcome with so much ease as to continue work for eight hours per day."

It may be noted that the first mention of horse-power in print occurs in Vol. II. of the first edition of the "Mechanics" of Olinthus Gregory, 1805.

Farey was a friend of James Watt, and a man of much ability, and moreover the inventor of an excellent form of slide rule, which appears to have been much used in the calculation of the dimensions of Watt's engines. Some of the slide rules were devised by Watt himself and one of his engineers, by name Southern. These instruments were known as the Soho sliding rule, and formed one of the everyday tools of the workmen employed by Watt (pp. 531—532 *idem*), as is evident from the following words:—"and having observed the facility with which the Soho workmen performed their ordinary calculations by it." I mention this, as I have heard some engineers speak of the slide rule as a new sort of instrument, and therefore one to be avoided. As a matter of fact the slide rule dates from a much earlier period, probably 1662 (see *Nature*, Vol. LXXXII.).

Experiments were made with much care by R. L. Edgeworth, F.R.S., on the relative work required to pull carriages of varied construction along ordinary roads. The results are collected in a volume entitled "An Essay on the Construction of Road Carriages," by R. L. Edgeworth, 1813. One experiment whereby the pull on two carriages carrying different loads was found deserves especial notice. The two carriages to be compared were pulled along a road by means of a rope attached to each, which passed round a horizontal pulley, the axle of which was fixed to another carriage in front, pulled by a horse. When the friction of each carriage was the same in value they advanced side by side, but when different, by loading, the friction of one of them was changed, one advanced on the other as both were pulled along (*ibid.*, p. 191). Two carts, one of which was furnished with springs, were tested; while the load of the cart having springs was as 15 to 12, the carts remained abreast of one another, thus showing the

advantage of the introduction of springs. The results are brought together in a paper by Mr. Ward, in the Third Report for 1809 of the House of Commons Committee upon Road Wheels and Roads.

A somewhat similar method has been used for comparing the friction of two boats drawn through still water, but in this case the relative friction was found by knowing the lengths of the arms of a lever, which took the place of the pulley in the former experiment. The author has also employed a modification of the same method, in which the horizontal wheel in Edgeworth's experiment was furnished with a Salter's balance, which indicated the difference of pull on the two bodies drawn along by the front carriage; in this way the relative resistance of two motor cars may be easily estimated—the two cars under comparison not using their own engines during the experiment. In many other experiments the same method of reading the difference of pull has been found to give excellent results—for example, in testing the surface friction between two pairs of similar surfaces, in one set the rubber moving in a straight line, in the other the rubber having in addition a small simultaneous lateral movement.

Coulomb investigated with great care the conditions under which men best performed *work* (Mémoires de l'Institut National des Sciences et Artes: Coulomb, "Science, Math. et Phys.," T. II.). His work is commented on by Poncelet in his "Mécanique industrielle": Paris, 1841, p. 237. The table exhibited by Poncelet, taken from the results of Coulomb, is most instructive, as it is a summary of his experiments, made with great care and extending over many years of observation. The following translation describes one of the most remarkable discoveries of Coulomb:—

"Concerning the best method for utilising the strength of man, the table shows that the greatest amount of work which a man can yield per day without undue fatigue consists in raising his own body, and this equals 280,800 kilogram-metre units of work in eight hours. Since the kilogram-metre = 7·233 foot-pounds, 280,800 kilograms = 2,031,026 foot-pounds in eight hours, or 4,231·2 foot-pounds per minute, so that the rate of doing *work* was 0·128 horse-power—a result at least seven times that of a simple worker with a shovel, and one which surpasses by

nearly two-thirds that of a workman employed in turning a winch handle. In order to utilise that amount of work at disposal there is no question, as Coulomb observes, but to make use of the descent of the weight of the man in raising a weight equal to his own to the height which he reaches each time, *i.e.*, each journey. Amongst the contrivances devised to fulfil this end the most simple, and that which has been practically used by Captain Coignet in the construction of the earthworks of the fort of Vincennes, near Paris, consisted of a rope passing over a pulley, furnished at its ends with boards, one of which carried the man and the other the weight to be raised. These operations, in which each workman raised the weight of his own body (70 kilograms) 310 times daily to the height of 13 metres, have been thoroughly authenticated."

The process will perhaps be better understood if we imagine a grooved pulley, running on a horizontal axle fixed at the top of a building, furnished with a rope having its lower end attached to the load to be raised. The workman ascended the building by steps, and when at the top he seated himself on a board attached to an upper portion of the rope, the load being a little less than the weight of the man; he then descended, regulating the rate of descent as he pleased, by handling the rising portion of the rope (which would be itself in equilibrium if continuous) until he reached the ground. Before he left his seat the load at the top would be removed, another man would take its place, and the man below who had made the descent would be replaced by the next load, which would ascend in the same manner. At the same time the first man would be walking up again to the top of the building, to be ready for the next load.

To-day one may see ships and colliers unloaded by men employing the same principle as that just described. A basket filled with coal by men in the hold of a collier is hauled up by means of a rope passing over a pulley suspended from a spar; the rope is furnished with numerous ends, four or five; each end is seized by one of a group of men standing on a platform. The men jump from the platform at the same moment, and their joint weight brings up the load: this is then upset down a shoot into a lighter or truck; the men then return to the platform for their next load. It is difficult to determine the date of this method of employing manual labour, but

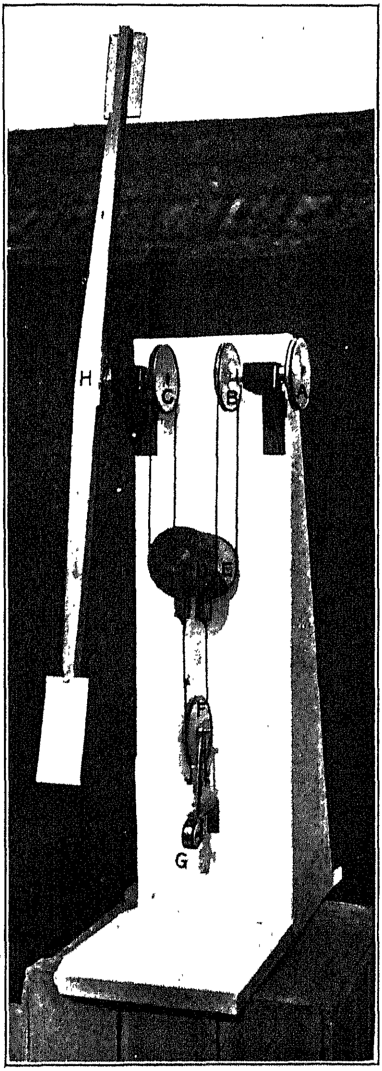

Fig. 2.

it is probably a very early one.

A very simple and not to be despised method of testing a prime-mover such as a gas engine or steam engine (the ports being open to the air) with a view to finding its internal friction is to set the engine in motion by means of a rope coiled on the flywheel, the rope being furnished with a weight sufficient to give the flywheel some convenient slow rotation. The weight may either descend into a dry pit, or, if high buildings are available, the rope may be taken over a pulley at such a height that the weight may have the required fall while rotating the flywheel of the engine slowly.

The method, which will be found instructive, can be easily applied for testing any small machine in which rotation takes place. The weight can be most conveniently applied and increased by means of an open cylinder or bucket attached to the cord, into which dry sand is poured from a hopper furnished with a sliding gate, by means of which the sand can be shut off at any instant. It is convenient to determine the weight of the cylinder, so that only the sand has to be weighed in each experiment.

The machine described by Hirn * appears to be the prototype of those work-measuring machines in which lines, ropes, or driving bands are employed in driving the machine or element of a machine to be tested. Two grooved pulleys are free to revolve on their horizontal axes, which are either in the same straight line or are parallel to one another. A continuous cord embraces half of each of the pulleys, and the two portions which hang down pass round pulleys, to the blocks of which scale pans are attached, which can be loaded as required. If one of the pulleys first mentioned be held fast, while the other is rotated, then one of the weighted pans will be raised and the other lowered. If one of the pulleys is connected to a fan, for example, the air resistance of which is sought, then when it is driven through the cord its tension is measured by half the difference of the weights in the scale pans, and the work done is deduced from the velocity of rotation and the tension of the cord. In Fig. 2 is shown a modification of this apparatus by the author, the two pans and weights in the apparatus of Hirn having being replaced by a pulley carrying a loaded arm G.

The pulleys A and B are fixed on the same axis, which runs on ball-bearings, A being driven by the prime-mover. The continuous band BDCE passes round the pulleys. The object to be driven, such as a wind vane, is fixed at H to the axis of C. To the sheaves of the pulleys D, E, a band is attached which passes round the pulley F and is fixed to it, so that when D and E are displaced the weight G, carried on an arm fixed to F, is raised, and from its position on a calibrated dial the difference of tension on the two sides of the driving band is known. This form of apparatus runs very steadily and is remarkably free from vibration. It has been found that good, strong fishing-line, carefully joined with a long splice, makes an excellent driving band when moderately small forces have to be dealt with. Line of this kind is largely employed in the Admiralty experimental ship model tank, and its excellence as a transmitter of motion has been constantly proved during a period of over fifty years that it has

* Bulletins de la Société industrielle de Mulhouse, 1854; and "Recherches Expérimentales sur la Relation qui Existe entre la Resistance de l'Air et sa Temperature," par G. A. Hirn, Acad. Royale de Belgique, 2 Juillet, 1881.

been employed elsewhere in connection with model ship-testing apparatus.

In dynamometers of the above-mentioned type it has been suggested that there is some difficulty in calibrating the machine, arising from the idea that the effective diameter of a V-grooved pulley carrying the cord could not be truly determined. Such a measurement may be difficult to make, but it is not really required. What must be known is the work done due to the difference of the tensions on the two sides of the driving cord, or band, multiplied by the space through which this difference of the tensions acts. If while a known weight is raised at a uniform velocity the difference of the scale-pan loads, or the reading of the pointer in the author's form of this machine, be known, then the constant of the ergometer can be found at once. Let this difference of the loads or the pointer's movement be automatically marked on a drum covered with paper, rotating at a speed proportional to the distance through which the force acts, then the area of such a diagram, multiplied by a suitable constant, shows the whole work done. For an illustration of this method see chapter on Planimeters.

Before leaving the gravity method a modification of its application must be considered, namely, that in which a weight falling through a known height is made to do *work*. In the hands of Joule the gravity method of measuring *work* was turned to admirable account in his determination of the mechanical equivalent of heat.* The paper was read before the Royal Society on June 21, 1849, but before that date weights falling through a measured height while rotating a bar of iron under magnetic influence were used by Joule to determine the *work* done in heating the bar. He states the result obtained thus :—" Therefore the heat evolved by a revolving bar of iron is proportional to the square of the magnetic influence to which it is exposed." A vertical axle, by which the iron bar was rotated, was driven by means of a double strand of fine twine carried over two easily-working pulleys placed on opposite sides of the axle. By means of weights placed in the scales attached to the end of the strings

* The Scientific Papers of James Prescott Joule, F.R.S., published by the Physical Society of London, 1884. See *ibid.*, p. 150, and p. 298.

the force necessary to move the apparatus was easily ascertained. In the research on the mechanical equivalent of heat, the same method of driving a vertical spindle connected to paddles by means of which water was churned in a copper vessel and thereby heated was employed, the fall being 63 inches and the weights either 29 pounds or 10 pounds a-piece. In order to obtain sufficient *work* for churning the water the weights were repeatedly raised, and allowed to fall, through the same height. The sum of all the heights of fall multiplied by the weights equalled the whole work done in any experiment in which a given weight of water was heated. The following correction was made: the weights reached the ground with a velocity of 2·42 inches per second. This was due to a height of 0·0076 inch and was subtracted from the fall in each case, giving the correct dynamic height fallen through.

The author is indebted to the following gentlemen for their kindness in sending him original papers, drawings, and automatic records of work-measuring machines, and he wishes here to thank them for their excellent assistance.

He also wishes to thank those institutions and societies which, through their several secretaries, have given him permission to reproduce paragraphs and figures from their transactions and papers.

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CHAPTER II

FRICTION

SINCE in work-measuring machines of the absorption kind, in which energy received by the machines is converted into heat by means of friction set up between rubbing elements, either solid, as in the band ergometer, or liquid, as in the machine of the late W. Froude, F.R.S., the laws of friction and certain aspects of its application demand our attention. It is not my purpose to give here more than a sketch of the subject; for exhaustive information respecting friction the following authors * may be consulted.

* Coulomb, Memoir by, 1785; Rennie, Phil., Trans. Royal Soc., 1829; Morin, Memoir, French Acad., 1831–34; "Nouvelles Expériences, etc., Faites," à Metz,

Up to the year 1870 the doctrine taught respecting friction was embodied in three laws, namely :—

(1) The magnitude of the frictional resistance between a given pair of surfaces of any materials is proportional to the pressure that keeps them in contact.

(2) The frictional resistance is unaffected by the area of contact.

(3) The frictional resistance is wholly unaffected by the relative velocity of the rubbing surfaces. Subsequent research has shown that the third law, as given by Willis in his "Principles of Mechanism," is not borne out by experiment when applied to journals running in bearings at velocities varying between wide limits.

We are greatly indebted to Hirn, Beauchamp Tower, Thurston, and O. Reynolds for their exhaustive researches on the friction between pairs of elements moving over one another at different velocities.

For the laws of friction due to a surface moving in water we are indebted to the late W. Froude, F.R.S. ("The Fundamental Principles of the Resistance of Ships," Proceedings of the Royal Institution, London, May 12, 1876). See page 139.

A statement of the laws of friction, as now received, between solids and a comparison of these laws with those of fluid friction are admirably stated by Prof. J. Perry, F.R.S., in his "Applied Mechanics," p. 80, 1909. See next page.

As possibly some of the readers of this book may wish to make experiments on friction, I now describe a simple method whereby the relationship between the load moved along a plane and the force required to move it may be discovered.

1834; Hirn: his results are collected in "Introduction à la Mécanique Industrielle," J. V. Poncelet, 3rd ed., Paris, 1870; Poncelet, "Introduction à la Mécanique Industrielle," Paris, 1870; Ball, "Experimental Mechanics," 1871; Thurston, "Friction and Lubrication," New York, 1879; Galton, *Engineering*, Vol. XXV.; Beauchamp Tower, Proc. Inst. Mech. Engs., 1883–5; Jenkin and Ewing, Phil. Trans., Vol. CLXVII., Part II.; Moseley, "Mechanical Principles of Engineering"; Reynolds, O., Collected Papers, pub. Cambridge University, and Ency. Brit., Vol. XXX., p. 372; Perry, J., "Applied Mechanics," Cassell, London, 1897.

For friction in connection with belt gearing, see Trans. American Soc. Mech. Eng., Vol. VII., p. 347, containing researches of Prof. Lanza, also the same society, Vol. VII., p. 549; Lewis, Vol. XV., p. 204; F. W. Taylor should be consulted. See, too, the admirable treatment of the subject of belt and rope gear in "Elements of Machine Design," Part I., by W. C. Unwin, F.R.S.; and Report and Observations on the Lille Experiments upon . . . Ropes for the Transmission of Power, by Prof. Capper, 1895, Inst. Mech. Engs., London.

Professor Perry's Comparison between Solid and Fluid Friction.

Friction between Solids.	Fluid Friction.
(1) The force of friction does not much depend on the velocity, but is certainly greatest at slow speeds.	(1) The force of friction very much depends on the velocity, and is indefinitely small when the speed is very slow.
(2) The force of friction is proportional to the total pressure between two surfaces.	(2) The force of friction does not depend on the pressure.
(3) The force of friction is independent of the areas of the rubbing surfaces.	(3) The force of friction is proportional to the wetted surface.
(4) The force of friction depends very much on the nature of the rubbing surfaces, their roughness, etc.	(4) The force of friction at moderate speeds does not much depend on the nature of the wetted surfaces.

Experience shows that almost any experiment carefully made to discover some physical condition is the most valuable way of acquiring knowledge about it. It is certainly far the most lasting in the memory.

When two surfaces are in contact, such as a book resting on a table, it will be found that if the book is made to slide along the table a certain force, acting parallel to the surface of the table, must be applied to the book in order to maintain uniform motion. This resistance to motion which is experienced is due to the force of friction. In order to obtain a numerical value of this force some such apparatus as that which will now be described may be used.

In the apparatus usually employed for showing the laws of friction the moving force is measured by some mechanism external to the body moved; this introduces a small unknown amount of friction due to itself, but for this a correction can be made. A plank of deal or any other suitable wood, about 5 feet $\times$ 9 $\times$ 2 inches, furnished with two ribs of wood fixed to its under side so as to support it from sagging, is carefully planed and surfaced. The sliding piece (which will be called the slide) is made of the same kind of wood 8 $\times$ 8 $\times$ 2 inches.

A cord fixed to an end of the slide passes over a pulley; to its end, which hangs down, a cylinder open at the top is suspended. Into this cylinder, which should be 12 inches long and 3 inches in diameter, dry sand is allowed to run in a fine stream; when the weight of the cylinder and sand has imparted to the slide a steady, slow motion, the flow of sand is stopped and the cylinder and sand weighed. The weight of the cylinder should be found and marked on it so that only the sand has to be weighed in each of a set of experiments. In each experiment the weight of the slide W must be altered; it is convenient to make the minimum weight of the slide, say 4 pounds, and then to add 4 pounds for each new experiment. The moving weight F must also be found in pounds and decimals of a pound. It will be found that the ratio of F to W is *approximately* a constant quantity. How, then, can we obtain a trustworthy value for the quotient $\frac{F}{W}$, which equals the coefficient of friction? This is usually denoted by the symbol μ.

The answer to this question is: A large number of experiments must be made and tabulated, and the best value found by the method of least squares, or by a graphic method which will be described.

In eight experiments the following numerical values were found :—

TABLE I.

No. of Experiment.	R Total Load on Slide in pounds.	Corrected Mean Value of Friction.	F Calculated Value of Friction.	Difference of Obs. and Cal. Values.
1	14	4·7	5·0	+ 0·3
2	28	8·2	8·5	+ 0·3
3	42	12·2	12·0	− 0·2
4	56	15·8	15·6	− 0·2
5	70	19·4	19·1	− 0·3
6	84	23·0	22·6	− 0·4
7	98	25·8	26·1	+ 0·3
8	112	29·3	29·7	− 0·4

METHOD OF LEAST SQUARES.

[" A number of observations being taken for the purpose of determining one or more unknown quantities, and these observations giving discordant results, it is an important problem to determine the *most probable* values of the unknown quantities. The method of least squares may be defined to be that method of treating this general problem which takes as its fundamental principle that *the most probable values are those which make the sum of the squares of the residual errors a minimum.*" This is the opening paragraph in the chapter on The Method of Least Squares in "Spherical and Practical Astronomy," Vol. II., p. 469, by Chauvenet. It is a consequence of the theory of probability that if a very large number of observations are made with a view to find some unknown quantity a more probable value of this quantity is obtained if the sum of the squares of the errors in the several observed results is made a minimum than if merely the algebraic sum of the errors is made zero. The latter corresponds to the arithmetical mean of the observations. It should be remembered, however, that the method of least squares is only properly applied where the number of observations is very great. With a small number only the use of the method is undesirable, in fact, it is merely a waste of time.]

If K is the coefficient of friction, we have to find the *best* value of K from sets of experiments made with different loads, which, when put into the equation $F - KR = 0$, will make $F - KR$ as close in value to 0 as possible. If R_1, R_2, R_3, R_m, etc., are the loads on the slide, and F_1, F_2, F_3, F_m the forces which act in each case, then we have to find a value of K, which makes

$$u^2 = (F_1 - KR_1)^2 + (F_2 - KR_2)^2 + (F_3 - KR_3)^2 \ldots + (F_m - KR_m)^2$$

a minimum. Let u^2 equal this sum.

$$u\,du = 0 = -R^1 (F_1 - K_1R_1)\,dK_1 - R_2 (F_2 - K_2R_2)\,dK_2 \ldots - R_m (F_m - R_mR_m)\,dK_m$$

equating to zero the coefficients of dK_1, dK_2 . . .

$$\therefore \quad K_1 = \frac{F_1R_1}{R_1{}^2}, \qquad K_2 = \frac{F_2R_2}{R_2{}^2} \ \ldots \ K_m = \frac{F_mR_m}{R_m{}^2}$$

$$\therefore \quad (K_1R_1{}^2 + K_2R_2{}^2 + \ldots + K_mR_m{}^2) = \Sigma F_1R_1,$$

taking K as the mean value, $K = \dfrac{\Sigma F_1R_1}{\Sigma R_1{}^2}$.

For convenience of calculation let the loads R_1, R_2, R_3, . . . R_m, have the values W, 2W, 3W, . . . mW,

so that
$$\Sigma F_1R_1 = WF_1 + 2WF2 + 3WF_3 + \ldots + MWF_m$$
$$= W\ (F_1 + 2F_2 + 3F_3 + \ldots + MF_m$$
and
$$\Sigma R_1{}^2 = W^2 + (2W)^2 + (3W)^2 \ldots + (MW)^2$$
$$= W^2\ (1 + 2^2 + 3^2 + \ldots + m^2)$$
$$= W^2 \frac{m\ (m+1)\ (2m+1)}{6}$$
$$\therefore\ K = 6\,\frac{F + 2F_2 + 3F_3 + \ldots + mF_m}{Wm\ (m+1)\ (2m+1)}.$$

Taking the values from the table,*
$$m = 8,\ W = 14.\quad F_1 + 2F_2 + 3F_3 + mF_m = 770{\cdot}9.$$
$$\therefore\quad K = 0{\cdot}27.$$

It has been found that an equation of the form $F = x + yR$ exhibits the results of values found by experiment more accurately than the former equation, $F = kR$. It may be established thus.

We cannot find values of x and y, such that the equation will be satisfied for all values of F and R, taken in pairs. But by the doctrine of least squares we know that the best values of x and y will cause the value of
$$(F_1 - x - yR_1)^2 + (F_2 - x - yR_2)^2 + \ldots + (F_m - x - yR_m)^2$$
to be a minimum; let it $= u^2$. This expression must be differentiated with respect to x and also y and the differential coefficients equated to zero.

$$u^2 = (F_1 - x - yR_1)^2 + (F_2 - x + yR_2)^2 + \ldots + (F_m - x - yR_m)^2$$
$$udu = 2\ (F_1 - x - yR_1)\ (-dx - R_1dy)$$
$$+ \ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$+ 2\ (F_m - x - yR_m)\ (-dx - R_mdy)$$
equate to zero coefficients of dx and of dy.
$$\therefore\quad \Sigma(F - x - yR) = 0 \text{ and } \Sigma R\ (F - x - yR) = 0.$$
Take
$$R_1 = W,\ R_2 = 2W\ \ldots\ R_m = mW,$$
then
$$F_1 - x - yW = 0$$
$$F_2 - x - y\,2W = 0$$
$$\ldots\ldots\ldots\ldots$$
$$Fm - x - ymW = 0$$
$$F_1 + F_2 + \ldots + F_m - m\,x - yW\ (1 + 2 + 3 \ldots + m) = 0$$
$$\text{Call } F_1 + F_2 + \ldots F_m = A,$$
then
$$A - mx - \frac{m\ (m+1)}{2}\,yW = 0\,. \qquad (1)$$

* This was constructed from the data of eight experiments, Table I.

also
$$WF_1 - xW - yW^2 = 0$$
$$2WF_2 - 2xW - y(2W)^2 = 0$$
$$\cdots\cdots\cdots\cdots\cdots$$
$$MWF_m - mxW - y(mW)^2 = 0$$
$$\text{Call } F_1 + 2F_2 + \ldots MFm = B,$$
then
$$W\,B - m\,\frac{m+1}{2}\,xW - \frac{m\,(m+1)\,(2m+1)}{6}\,W^2y = 0$$
$$B - \frac{m\,(m+1)}{2}\,x - \frac{m\,(m+1)\,(2m+1)}{6}\,Wy = 0 \quad . \quad (2)$$

From equations (1) and (2) we find
$$x = \frac{2+4m}{m^2-m}\,A - \frac{6}{m^2-m}\,B$$
$$y = \frac{12}{m^3-m}\,\frac{B}{W} - \frac{6}{m^2-m}\,\frac{A}{W}.$$

In making the experiment care should be taken to make the load on the surface of the slide equal to W, 2W, 3W, and so on. This simplifies the calculation, and enables us to sum the numbers $1 + 2 + 3 + m$ and also the squares of these numbers by means of known formulæ.

The method of finding x and y in the equation
$$F - x - yR = 0$$
has now been demonstrated, so that with this equation we have friction = a constant + coefficient of friction × load. By taking values of R and F from Table I. the equation becomes
$$F = 1{\cdot}44 + 0{\cdot}252\,R,$$
but it must only be applied to weights which lie within the limits of the values found by the experiments and shown in Table I.

I am indebted to Sir R. Ball, F.R.S., for the table taken from his excellent book on "Experimental Mechanics" (Macmillan) and the matter respecting the method of least squares, which has been slightly altered in order to show each step in the formation of the equations.

Diagram on Squared Paper.

If we wish to exhibit the values given in Table I. graphically we proceed thus (squared paper divided in tenths of inches will be found suitable for the purpose). Along the line OR, Fig. (3), we set off the values of the loads R on the slide and their

corresponding ordinates, showing the values of F for each value of R. The points on the squared paper at the top of each ordinate should be marked with a dot surrounded by a very small circle. The circle makes the next operation easier to perform. A fine black thread is stretched from the extremities of a small bow of wood or whalebone and placed so as to take a mean position amongst the dots; the point at which the thread cuts 0F is marked; its distance from 0 shows the value of the constant in the equation, and the tangent of the inclination of the thread to 0R gives very approximately the value of the coefficient of R; so that by means of a carefully constructed diagram we can exhibit the relationship of R and F, and it becomes a descriptive picture of the equation connecting the variable quantities found by experiment. For instance, the equation $F = 1{\cdot}4 + 0{\cdot}252R$ is represented by the straight line of the diagram, if we take $R = 70$, $F = 19{\cdot}0$ from the equation.

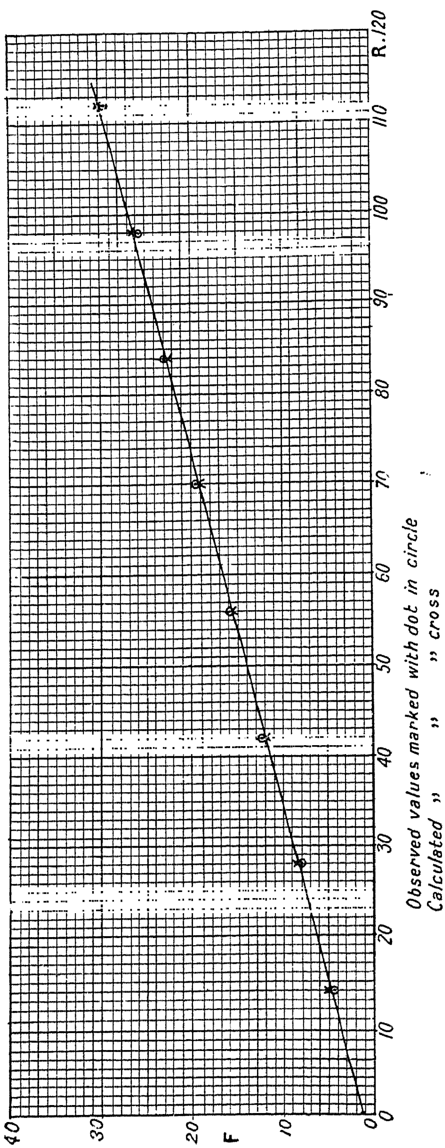


FIG. 3.

And taking 70 on the line of loads on the diagram we find $F = 19 +$ some very small quantity.

Almost innumerable examination papers on mechanics have been set and answered without any allusion whatever being made to friction ; and the impression left on the mind is that the papers must have been the product of those who must regard friction in mechanics as of but little importance. Probably friction has been omitted because it would introduce a little more difficulty into the solution. But the practice of neglecting it is simply vicious, since in every real, material mechanical contrivance friction exists, either for good or evil, and should be thoroughly appreciated by the student of mechanics and the engineer. "Friction is, so far as we are concerned, quite as essential a law of Nature as the law of gravity," writes Sir R. Ball.

Many of the examination papers, if somewhat remodelled, become interesting—that is, after friction has been correctly introduced—and the weightless cord has been replaced by something which really exists. A single question containing, as far as is known, the real conditions of the case is better worth answering than a thousand in which the many inseparable conditions are *neglected*.

[Where the number of observations is so small as eight, as in the instance last considered, even where the form of the equation connecting the observation and the calculated result is accepted—in this instance $F = kR$ or $F = x + yR$, as the case may be—the graphical method gives all the information which can properly be derived from the experiments, provided only that the scale is such that the uncertainties of observation when making the experiment are considerably greater than the uncertainty in the comparison on the diagram of the position of the points representing individual observations and those where the line representing the accepted law cuts the corresponding ordinates. Any apparent increased accuracy resulting from pushing the method of least squares to an extreme is entirely fallacious. If, as in the example here given, the observations are not numerous and the conditions are constantly changed so as to cover a large range in the values of the abscissæ, the graphical method is greatly to be preferred to any treatment of the figures by the method of least squares

in relation to a particular law, for, if, as in this instance, the law is empirical and not a law of Nature in itself absolutely true, the distribution of the points representing observations with respect to the line representing the empirical law themselves throw some light upon the propriety of accepting the law. We have already seen that the simple and imaginary law of friction $F = kR$ does not agree with experiment so well as the less simple law $F = x + yR$, but it cannot be inferred from this that this more accurate law is itself a true law. The diagram (Fig. 3) at once shows that the second law is a better representation of the fact than the first, but the "errors," that is, the distance of the points within circles from the straight line, have a certain consistent character. For abscissæ below 35 and above 90 they are below the straight line, while between these values they are above. With so small a number of observations it is not possible to assert that these divergencies indicate a truer law than that represented by the straight line. If, however, a repetition of the experiments consistently showed corresponding "errors," then the conclusion would be that a more complex law would be needed to represent the truth. Thus, if the graphical method is used on a scale large enough as already defined, not only may the constants relating to an assumed law be determined, but the propriety of the law may be ascertained, both in a minimum of time.

Where the law to be represented on squared paper is not of the form $y = a + bx$ it cannot be directly represented by a straight line. By the use of specially-ruled paper other laws may be represented by lines either straight or more simple in character than those which would be required upon paper ruled in equal squares. The most useful special ruling next to that of equal squares is the logarithmic ruling; that is, the distances on the paper are proportional to the logarithms of the numbers attached to the corresponding lines. Such paper may be bought already ruled or it may be extemporised by copying the distances from an ordinary slide rule and marking the main lines 1, 2, 3, etc., up to 10, after which it is unnecessary to repeat the ruling. On this paper (see Fig. 57, page 118) the law $y = ax^n$ can be represented by a straight line making an angle of $\tan^{-1} n$ with the axis of x. If n is negative as in the case of Boyle's law, the

straight line slopes downwards instead of upwards—at an angle of 45 degrees in this case. All such lines represent x^n, and the particular line is determined by the constant a. Another system of ruling (Fig. 4) with one set of lines spaced equally and the other logarithmically enables one to represent compound interest growth or logarithmic relations as a straight

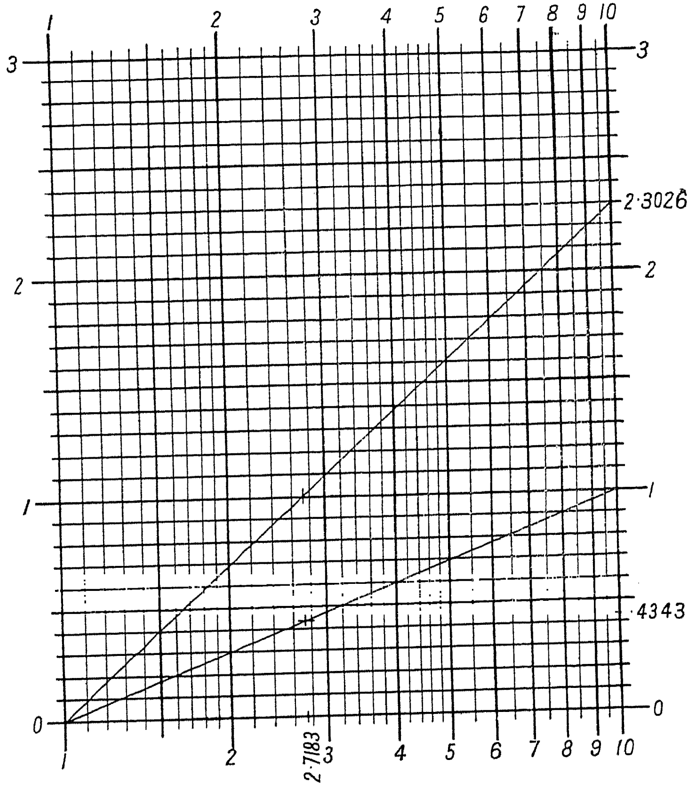


FIG. 4.

line. In Fig. 4 only one line in every 10 of the slide rule is drawn, to avoid confusion on the reduced scale. On ruling any straight line on such paper the value of the series of points on the equally-spaced lines are in arithmetical progression, while those on the logarithmically-spaced lines are in geometrical progression. Any straight line ruled through the point 0 on the equally-spaced set and 1 on the logarithmically-

spaced set provides a table of logarit`ms to some base. If it also pass through the corresponding point 1, 10 on the two scales the table of logarithms is to the base 10, for the logarithm of 10 to the base 10 is 1. Also the logarithm of e, the value of which is 2·7183, to the base 10 = ·43429 the modulus. If the tangent of the inclination is increased in the ratio of 1 : 2·3026, so that the line passes through the point 2·3026, 10, then the table of logarithms is to the base e, for $\log_e 10$ is 2·3026. Also the logarithm of e to the base $e = 1$. Log 1 = 0 according to any system, so all these lines must pass through the point 0, 1.

If in experiments on the friction of ropes or belts round drums, referred to in the next few sections, the angles, however measured, are plotted on the scale of equal parts and the observed pairs of tensions on the logarithmic scale, the line joining the corresponding points should be a straight line according to the theory as there expounded, and the agreement or otherwise of a number of experiments with one another can be ascertained immediately by inspection with an accuracy greater than that of the experiments. In plotting the pairs of tensions the smaller should be plotted as 1, and the ratio $\frac{T_1}{T_2}$ should be plotted as the other. The straight line passing through the point 0, 1, and as evenly as possible through the plotted points, represents the growth of the tension in the band. The quantity $\mu\theta$ may be read directly from the line giving logarithms to the base e, for this line cuts the line representing $\frac{T_1}{T_2}$ on the logarithmic scale at the point where the scale of equal parts shows the value of $\mu\theta$, and thus μ may be determined, but for this θ must be taken in radians, not in degrees, half-turns, or other arbitrary measure.]

The value of the friction of a band on a cylinder at any point in the arc of contact may be shown graphically by Prof. J. H. Cotterill's method as follows.

Let PQ (Fig. 5) be a section through the cylinder perpendicular to its axis, and let AB be an element of the band which embraces it, also let T_1, T_2 be the tensions on each side of the element, let the value of $T_1 - T_2$ be such that the belt is on the point of slipping. Then AB is kept at rest by T_1 and T_2

and the reaction against the cylinder R. The three forces meet at the point D. On OA set off Oa to represent on any convenient scale the value of T_1, draw ab perpendicular to R cutting OB in b; then, since the sides of the triangle abO are perpendicular to T_1, T_2, R, they are proportional to T_1, T_2, R, and Ob represents T_2 and ab, R. If AB be made very small, ψ will be the angle of friction. Commencing the construction at M and continuing it to N, a curve is described the radius

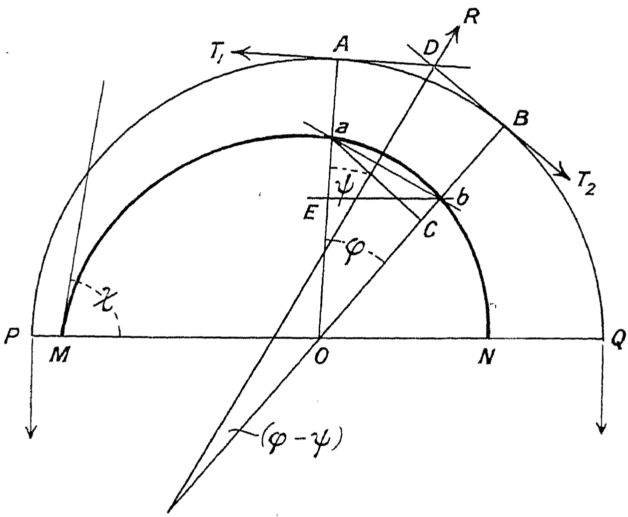


FIG. 5.

vector of which shows the value of the tension at any point. Also from a consideration of the diagram a useful formula may be constructed, thus :

Draw Ca, Eb, at right angles to OB and OA respectively, then

$$\frac{T_1}{T_2}=\frac{Oa}{Ob}=\frac{\sin Oba}{\sin Oab}=\frac{\frac{aC}{ab}}{\frac{bE}{ab}}=\frac{\cos Cab}{\cos Eba}=\frac{\cos(\varphi-\Psi)}{\cos\quad\Psi}$$

$$=\cos\varphi+\sin\varphi\tan\Psi.$$

If φ diminish indefinitely, $\cos\varphi$ becomes $=1$, $\sin\varphi=\varphi$ and $\frac{T_1}{T_2}=1+\varphi\tan\Psi$, or $\frac{T_1-T_2}{T_2}=\varphi\tan\Psi$.

Let $T_1 - T_2 = dT$ in the limit, and $\varphi = d\theta$,

then $$\frac{dT}{Td\theta} = \tan \Psi = \mu \text{ the coefficient of friction.}$$

This on integration gives $\frac{Tp}{Tq} = e^{\mu\theta}$,

where T_p and T_q are the tensions at P and Q respectively and θ is the angle in radians between OP and OQ. For a circular pulley the curve MaN is an equiangular spiral.

Mechanical Method of Drawing the Logarithmic or Equiangular Spiral (by the author).

The equation to the curve is $r = a^\theta$,

where r is the radius vector,

a is a constant on which the form of the curve depends,

θ is an angle swept out by the radius vector,

χ is an angle which the tangent to the curve at any point makes with the radius vector; it equals $\tan^{-1}\frac{1}{\log_e a}$.

In the logarithmic spiral the angle at the pole increases in an arithmetic ratio, while the radius vector increases in a geometric ratio; so that the angle generated by the radius vector is proportional to the logarithm of the length of the radius vector. The spiral is called equiangular, because the tangent at any point in it makes a constant angle with the radius vector. I have taken advantage of this property of the curve, and embodied it in the instrument now to be described.

A surface such as a drawing-board is covered with paper which receives the trace made by a small sharp-edged wheel. This wheel rotates in a forked bearing, its axis being parallel to the plane of the paper. The plane of rotation of the wheel may be placed at an angle to the longitudinal axis of the radius bar. This radius bar is free to slide through a vertical axis, taking the form of a small pillar erected on the board. When the radius bar rotates about the pillar and the plane of the wheel is at right angles to it, the path of the wheel is a circle, but if the plane of the wheel be set at some angle χ less than 90 degrees to the radius bar, it will traverse over the equiangular spiral, for it moves continuously tangential to its path.

Clear traces may easily be obtained by placing carbon paper over the paper on which the trace is required and weighting the wheel enough to make it mark the paper under it as it rolls.

Instruments of the kind described, made from my designs, have been used by me in the Engineering Laboratory, Oxford, for showing the growth of the curve $r = a^\theta$, and one was added in the year 1886 to the collection of apparatus in the Physical Laboratory of Winchester College so admirably organised by Mr. W. B. Croft.

In order that the instrument may be applied to an experiment on the friction of a belt on a pulley, the wheel (called the tangent wheel) must be set at the angle χ to the radius vector. To do this we must know the values of T_2 and T_1 and the angle θ between the two radii vectores r_2, r_1, which are proportional to T_2 and T_1. Let the angle θ be measured in radians.

$$\text{Then } \tan\chi = \frac{\theta \log_{10}e}{\log_{10}r_2 - \log_{10}r_1}.$$

For example, suppose the belt to make half a turn on the pulley and the tension of one end to be double that of the other, or in symbols—

let $\theta = \pi = 3{\cdot}1416$

let $\dfrac{r_2}{r_1} = 2$

$\log_{10}e = 0{\cdot}43429$

then $\tan\chi = 4{\cdot}5324$ and $\chi = 77^\circ\ 33'\ 29''$,

and since $r = a$ and $\tan\chi = \dfrac{1}{\log_e a} = \dfrac{\log_{10} e}{\log_{10} a}$,

$$\log_{10} a = \frac{\log_{10} e}{\tan\chi} = \frac{{\cdot}43429}{4{\cdot}5324} = {\cdot}095818,$$

whence $a = 1{\cdot}2469$ and the equation becomes

$r = 1{\cdot}2469^\theta$;

when $\theta = o$, $r = 1$, when $\theta = \pi$, $r = 1{\cdot}2469^\pi = 2$.

[The tangent wheel is to be set so as to make an angle of $77^\circ\ 33'\ 29''$ with the radius vector, or as near that as possible, and the instrument made to trace the curve for an indefinite number of turns. It will then be found that if any straight line be drawn through the pole, at which point the pillar is fixed, it will cut the spiral in a number of points, and that the ratio of the distances from the pole of any two such consecutive

points on the curve is as 2 to 1, agreeing with the assumed condition that the belt should double in tension in one half-turn.

$$\text{Since } \frac{T_1}{T_2} = \frac{r^1}{r_2} = e^{\mu\theta} = 2$$
$$\mu\theta = \mu\pi = \log_e 2$$
$$\mu\pi = \cdot 69314$$
$$\mu = \cdot 221.$$

So ·221 is the coefficient of friction under which a belt doubles in tension for every half-turn that it makes.

The evaluation of μ when $e^{\mu\theta}$ is known is tiresome rather than difficult. This can be effected with abundant accuracy by means either of the semilogarithmic chart (Fig. 4), already described, or of the slide rule with a log log line, invented early last century by Dr. Roget and reinvented several times since, or the exponential curve of Fig. 7 may be used. Calling the log log line of the rule the P line and one of the ordinary log lines the C line, it is merely necessary to set $\log_e 10$ or 2·3026 on the C line opposite 10 on the P line, then opposite $\frac{T_1}{T_2}$ or $\frac{r_1}{r_2}$ or the ratio of the tensions of the ends of the belt on the P line will be found $\mu\theta$ on the C line. This product can be divided by θ, or in the last instance by π, on the A and B lines or upon another rule, and thus μ may be found very quickly. Where a large number of experiments have to be reduced this is preferable to the usual treatment with logarithm tables, as the whole series of values of $\mu\theta$ can be read from a single setting of the rule, and the accuracy is sufficient. The semilogarithmic chart is almost equally convenient.]

Automatic Friction Machine.

The Relationship between the Diameter of a Pulley and the Coefficient of Friction of a Band in Contact with the Pulley.

It has been shown by the experiments of Imray that with pulleys widely differing in diameter the coefficient of friction is nearly the same.

His experiments were made thus: a pulley was held fast on a horizontal axle, a belt embraced half its circumference, and weights P W were suspended from its extremities. The

weight W was gradually increased, until the belt just began to slip; the value of (W − P) equalled the frictional resistance between the pulley and the belt. From data so obtained the ratio $\frac{W}{P}$ was determined: if then when the arc of contact embraced by the belt is constant the ratio $\frac{W}{P}$ remains invariable,

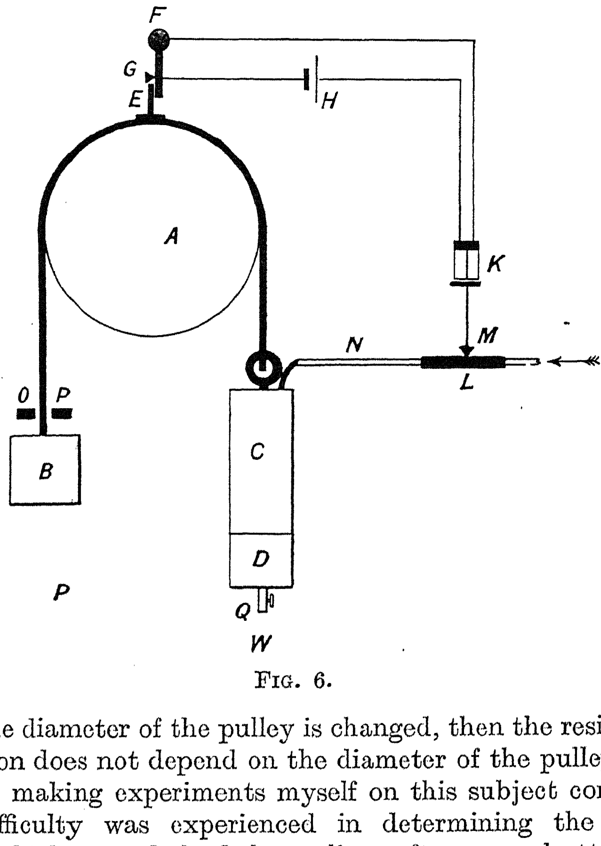


FIG. 6.

when the diameter of the pulley is changed, then the resistance of friction does not depend on the diameter of the pulley.

When making experiments myself on this subject considerable difficulty was experienced in determining the exact weight which caused the belt to slip; after several attempts, the following plan was devised by me, whereby the value of W at the instant of slipping was automatically recorded. The arrangement of the apparatus is shown in the diagram (Fig. 6), in which A is the fixed pulley and B D the two weights suspended from its ends. The weight D is furnished with a

hollow cylindrical vessel C of known weight w; the vessel is filled with water from the tube NM; the water can be instantly stopped by the edge of a metal plate M when released by an electromagnet K on to a piece of rubber tube L.

The instant the belt begins to slip the projection E touches the electrical contact piece F; it is released from the contact G and the current from the battery H is broken; the supply of water through MN ceases, so that if the water in the vessel be drawn off at Q and weighed the exact difference of the pulls on the two sides of the belt may be ascertained.

To return to Imray's experiments: five pulleys were used of small diameter, ranging from 5·5 inches to 14 inches; the rounding of the face of the pulley was about 0·125 inch in 2 inches; the surface was polished, and the same belt was used in each experiment, being oily and pliant; its width was 1·62 inches and its thickness 0·12 inch. The results obtained are embodied in the following table:—

TABLE II.

No. of Experiment.	Diameter of Pulley, inches.	Weight of P, in lbs.	Weight of W, in lbs.	Value of $\frac{W}{P}$.	Mean Value of $\frac{W}{P}$.
1		18	29	1·611	
2	5·5	34	57	1·676	1·675
3		65	113	1·738	
4		17	29	1·706	
5	7·6	32	57	1·781	1·733
6		66	113	1·712	
7		17	29	1·706	
8	9·8	32	57	1·781	1·733
9		66	113	1·712	
10		18	29	1·611	
11	11·8	34	57	1·676	1·642
12		69	113	1·638	
13		17	29	1·706	
14	14·0	34	57	1·676	1·673
15		69	113	1·638	

Average value of $\frac{W}{P} = 1·691$.

The experiments show that the friction in the case of the largest pulley was almost the same as that of the smallest

one ; from the agreement of these fifteen experiments it may be inferred that the amount of friction is not affected by the diameter.

In the formula for the friction of a belt on a pulley, namely, $\log_e \frac{W}{P} = \mu\theta$, in which W and P are the two loads, μ the coefficient of friction, and θ the angle in radians embraced by the belt, and $e = 2{\cdot}71828..$, the diameter of the pulley is not involved. When a driving-belt slips on a pulley owing to overload, the slip may be prevented by employing a pulley of greater diameter ; this does not *increase* the friction, but since the circumferential velocity of the larger pulley is greater

FIG. 8.

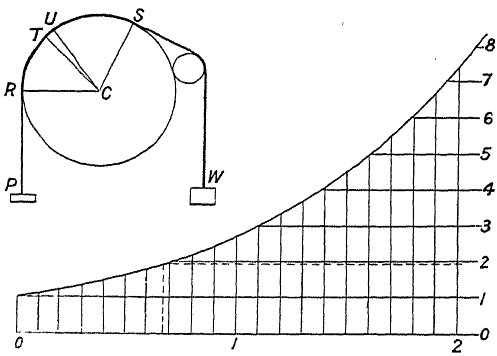


FIG. 7.

than the smaller one, the friction necessary to transmit the same power at the same number of revolutions per minute is proportionately less, and so may be reduced below the slipping limit.*

* [NOTE.—There is a useful limit to the size of a pulley in any case, determined by the centrifugal force of the belt itself, which diminishes the pressure of contact between the belt and the pulley and hence the friction. For a belt of any material there is a practical working limit to its longitudinal tension. If M is the number of pounds per foot run (mass per unit length) and V the linear velocity of the belt in feet per second, the longitudinal tension needed to balance the centrifugal force may be shown to be $\frac{Mv^2}{32{\cdot}2}$ pounds, and this is independent of the diameter of the pulley over which the belt runs. With such a tension only in the belt there is nothing available for friction, so the tension necessary for the frictional drive must be additional and it is only this additional tension which is effective. With increase of diameter of pulley the work due to a given effective tension increases in direct proportion, while the tension needed to balance centrifugal force increases in squared proportion. So when this is subtracted from the safe tension of the belt the work which can be transmitted, far from increasing indefinitely with the linear speed, is a maximum at some speed which depends on the ratio of the safe

This equation is exhibited diagrammatically in the curve Fig. 7, and the ratio $\frac{W}{P}$ can be found at once for different values of μ and θ. Suppose $\mu = 0{\cdot}33$ and $\theta = 2$, then $\mu\theta = 0{\cdot}67$; read this value along the base line, the corresponding ordinate $= 1{\cdot}95$, the value of $\frac{W}{P}$. The curve is drawn thus: the base line is divided into tenths of some unit up to the value 2. The product $\mu\theta$ is given values from 0 to 2. The curve drawn through the corresponding ordinates is the required one. The results of experiments on larger pulleys are exhibited in the following table :—

Table III.

Friction of Belts on Pulleys of large Diameter, with variable Arc of Contact.

No. of Experiment.	Diameter of Pulley. Inches.	Arc of Contact. Degrees.	Value of P in lbs.	Value of W in lbs.	Ratio of $\frac{W}{P}$.	Mean value of $\frac{W}{P}$.	Calculated Value of $\frac{W}{P}$.	Error per cent.
29			14	25	1·786			
30	15·8	120	42	84	2·000	1·962	1·938	$1\frac{1}{4}$
31			70	147	2·100			
32			14	24	1·714			
33	24·0	123	42	78	1·857	1·809	1·778	$1\frac{3}{4}$
34			70	130	1·857			
35			14	34	2·429			
36			16	39	2·437			
37			21	52	2·476			
38	38·8	144	28	70	2·500	2·506	2·491	$\frac{1}{2}$
39			42	109	2·595			
40			70	182	2·600			

The calculated value of $\frac{W}{P}$ was obtained by taking $\mu = 0{\cdot}316$, as deduced from the experiments tabulated in

working tension to the weight of the belt per foot. Thus leather belts should not run more than about 4,000 ft. per minute, hemp or cotton ropes about 6,000, while steel bands or ropes may run much faster.]

Table II., and reducing the arc in degrees of Table III. to circular measure θ, and putting these values into

$$\log_e \frac{W}{P} = \mu\theta.$$

The largest pulley appears to have given the greatest friction, but the experiments on the smallest pulley show more friction than those on the pulley of intermediate size. Such a discrepancy has been accounted for by the difference of polish of the surfaces. The result with any one of the pulleys does not vary from the average more than by a small quantity, which might be accounted for by the difference of polish mentioned. This appears to have been the opinion of Imray. In all belt friction experiments it is no easy matter to determine the exact amount of arc of the pulley in contact with the belt. Fig. 8 shows how the length of arc RS is regulated, by means of a small pulley on the right, carried on an arm free to rotate about C. The small pulley must not touch the large pulley, though it is made to do so in the figure.

Table IV.

Experiments with the Automatic Friction Machine (by the author). A cotton rope ($1\frac{1}{8}$ inch circumference) was used on a V-grooved pulley, well polished but not lubricated.

No. of Experiment.	Weight P.	Weight W.	Ratio $\frac{W}{P}$.	Mean $\frac{W}{P}$.	$\log_{10} \frac{W}{P}$	Diameter of Pulley V-grooved, in feet.
A 1	145·8	336	2·304	—	·3625	1
A 2	75·5	220	2·925	2·806	·4661	1
A 3	110	295	2·681		·4283	1
A 4	75·2	212	2·819		·4501	1
A 5	40	112	2·800		·4472	1
B 1	181·5	445	2·451	—	·3893	0·5
B 2	40	122	3·050	2·824	·4843	0·5
B 3	75	228	3·040		·4829	0·5
B 4	145·5	369	2·536		·4041	0·5
B 5	110·5	295	2·669		·4264	0·5

In experiments 1 A and 1 B. The result is affected by too great a load, and the rope was deformed in section. Experiments 2—5 A and 2—5 B give better results, and these only are taken to find the mean values.

The quantities W, P, μ and θ are connected together by the equation

$$\frac{W}{P} = e^{\mu\theta},$$

the generation of which has been already shown. In these experiments the rope embraced half the pulley so that $\theta = \pi$, $e =$ the base of the hyperbolic logarithms, and

$$\frac{W}{P} = e^{\mu\pi},$$

and, finding the value of μ from this equation and using the mean values in set A and set B, we find

$$\text{for set A, } \mu = 0{\cdot}328,$$
$$\text{for set B, } \mu = 0{\cdot}330.$$

Since in the experiment described the rope rested in a V-groove, if the radial force Q be taken as unity the whole normal pressure R between the sides of the V-groove and the rope is Q cosec φ, where $2\,\varphi =$ the angle of the groove, so that $R = Q \operatorname{cosec} \varphi$. And the resistance to slipping is

$$\mu R = \mu Q \operatorname{cosec} \varphi.$$

If $\mu\theta \operatorname{cosec} \varphi$ be written for $\mu\theta$ in the equation $\frac{T_2}{T_1} = e^{\mu\theta}$, we have an equation, $\frac{T_2}{T_1} = e^{\mu\theta \operatorname{cosec} \varphi}$, which is applicable to the case of a rope lying in a V-groove.

[An instructive experiment on the same subject requiring the minimum of apparatus may be made by means of an umbrella, a spring balance, and a piece of tape. Hang the balance from a bracket and fasten one end of the tape to the balance; pass it under the hooked handle of an umbrella and hold the free end in the hand so that both ends of tape are vertical. When raising the hand, the umbrella will appear to weigh less than it does when the motion is in the opposite direction. The sum of the two readings on the balance M will be found to be equal to the weight of the umbrella when suspended directly. The two readings give T_1 and T_2, and quite good observations may be made in this way.

The extremely rapid increase in the friction between a rope and anything round which it is wound as the angle of winding is increased is referred to in a subsequent chapter, more especially in relation to bollards and capstans. This principle is also made use of in drawing wire. It would be impossible to draw wire through a number of draw-plates in series by pulling at the end, for each draw-plate requires a force comparable with the breaking load of the thinner wire leaving it to draw the thicker wire through. As the wire leaving each plate runs at a higher speed than it did in entering the plate it would be impossible to give suitable movements to the parts between successive plates by any direct gripping device, for if the pull on any one of the finer parts of the wire leaving a plate were ever so little too rapid to correspond with the slower motion of the thicker parts entering the plate the wire would break instantly, whereas if it were too slow the wire would accumulate between the plates and become entangled. The whole difficulty is overcome by placing between each pair of plates a smooth revolving wheel round which the wire makes a turn. Only beyond the last plate is the wire wound up positively on a moving drum. All the intermediate wheels are made to turn at speeds somewhat in excess of those necessary at the corresponding points, but they only draw the wire through the preceding plate at the exact speed required to feed succeeding plates, for it is only at this rate that there is any tension on the wire leaving the wheel, and this is magnified in the ratio of 1 to $e^{2\mu\pi}$ for the one turn round the wheel.

A capstan or one of the wheels between a pair of draw-plates just described is really a mechanical relay. A smaller force applied to the end of the wire or cable leaving the capstan or wheel is magnified in a definite ratio, which ratio may be made small or very great as required by making the angle of wind moderate or great, as already sufficiently explained. It is a mechanical relay in two senses; either a force magnified in the desired ratio may be applied to a support yielding, for instance, under an elastic law, when the displacement of the attachment from its zero position will at all times be proportional to the applied force, or again, if the magnifying power is sufficient to overcome the greatest resistance to be met

with, the relay is of a different type, copying the movement but disregarding the opposition.

Mr. S. G. Brown* has made use of this principle in one of his submarine cable relays, where the signalling current received by the wires *a a* is insufficient in strength to cause the receiving coil A to move the relay contacts. The receiving coil (Fig. 9) is suspended by a fibre in a magnetic field N S, and is so supported that it can rotate through a small angle about a vertical axis. Two fibres *s* attached to the sides of the coil each take one or more turns round a drum D and then pass on to the arm of the relay R, to which they are fastened. The drum is kept turning towards the receiving coil, as shown by the arrow.

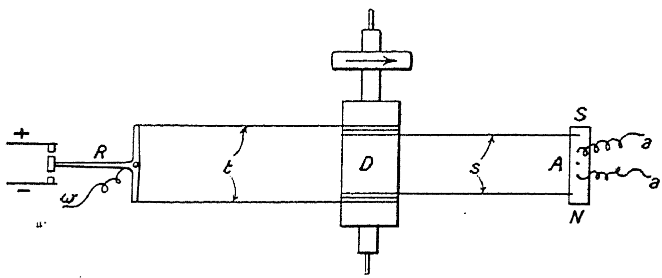


Fig. 9.

When a signal current is received the tension in one or other of the fibres *s* is greatly magnified in the relay end of the same fibre *t*, so the relay faithfully follows the movement of the coil A even though the forces are magnified more than three hundred times. Thus the wire *w* is made + or — in obedience to the signalling currents received by the coil A.

Dr. J. G. Gray has made use of this form of mechanical relay in the gyrostatically controlled air- and water-ships which were described by him at a meeting of the Physical Society of London held on May 8th, 1914.†

A similar arrangement with a known ratio of magnification would make the dynamometry of the work done in the most delicate instruments comparatively easy, or a succession of

* Proceedings of the Physical Society, 1913, p. 131.

† Proceedings of the Physical Society of London, Vol. XXVI., Part IV., June 15th, 1914.

such relays, each of a size and strength suited to the forces met with, would steer a ship if desired with no greater controlling force than that due to a signalling current.

In the author's manuscript I found the following memorandum, "Add note on Lateral Friction," but he does not appear to have left any such note. This curious branch of the general subject is one which interested him greatly. Lateral friction is best defined by reference to an experiment which the author showed to me, which exhibits the phenomenon in a striking manner. On a horizontal shaft which can be made to turn at any desired speed fix a smooth cylindrical pulley and pass over this pulley a pliable belt, with a weight at one free end and with the other end fastened to the floor, so that when the pulley is turned in either direction the belt will slip upon it. The pulley should be several diameters above the floor. When the pulley is revolving and the band is at rest it will be found that the slightest lateral force applied to the belt will cause it to shift its position on the pulley. If the pulley is turning sufficiently fast the response is instantaneous, and the lateral movement is limited only by the attachment of one end to the floor. Forces far below that necessary to overcome the friction of the belt on a stationary pulley are immediately evident.

The nature of lateral friction, or rather the reason for its absence, will be made clear by the consideration of an imaginary experiment with a block and an inclined plane. When the plane is inclined at an angle α below that which will cause the block to slide, the block may be pulled sideways by means of a thread, and then when the component of this lateral pull and W sin α due to gravity itself exceeds μW cos α, the block will move in the direction of this component force. Next suppose the block to be sliding down the incline very quickly and then a small lateral force to be applied by the thread, the block will then, as before, move in the direction of the component of the resolved force down the plane due to gravity and that applied by the thread, and this will be very slightly inclined to the direction of the greatest slope. Therefore, in proportion as the block is sliding the more quickly so will the lateral movement be the more rapid. As this experiment would be inconvenient to make, consider its equivalent when the block is supposed to be supported by and within a large hollow cylinder revolving

about a horizontal axis at a high speed. The block will take a position within the drum having an inclination α, such that W sin α is equal to the running friction. Then a slight lateral force will have the effect of producing an immediate and rapid lateral response, and the fact that the relative path of the block on the surface is only slightly inclined to its former path will not be evident. The lateral motion alone is visible. Thus the more rapid the motion of the surface the more rapid the lateral response. There is the further peculiarity that the lateral friction is non-existent, for however small the lateral force may be the component of this and that due to friction parallel to the surface will have some inclination, and so the block will move sideways at a speed which is the same fraction of the speed at which the surface is moving as that which the lateral force is to the frictional force.

This absence of lateral friction has been made use of in one of the forms of cable relay invented by Mr. S. G. Brown, in which a light pointer attached to a pivoted coil, through which the minute current from the ocean telegraph cable passes, rests very lightly on the surface of a polished silver drum kept in rapid rotation. The friction of the pointer, however lightly resting against a stationary drum, is vastly greater than the force available to move it, which is caused by the reaction between the minute current in the coil and the strong magnetic field in which it is placed, and so if the drum were at rest no message would be received. As, however, the drum is kept revolving at a high speed, the pointer moves laterally under the feeble stimulus of the cable currents and the response is instantaneous. We are not concerned here with the electrical actions set up in consequence of this movement or with the other features of this ingenious instrument. It supplies, however, an excellent illustration of the non-existence of lateral friction and of the instantaneous response to lateral forces, far too small to overcome the real friction.

There is no difference in the action just described and that on a stationary belt of a pulley revolving within it. This action, however, differs entirely from the case of a running belt moved by a lateral force, as by a belt-shifter. All that happens when a lateral force is applied to the belt approaching a pulley is a slight angular deviation in the direction of the

leading-in side, which does not slip on the pulley, but proceeds to trace out a helical path having the same angle. Thus if the belt is running fast a belt-shifter works easily and very quickly. It is useless to apply a belt-shifter to the belt where it leaves a pulley, but with a stationary belt over a slipping pulley it is indifferent whether the lateral force is applied on one side of the shaft or on the other. The contrast between a running and a slipping belt on the usual slightly convex pulley of a machine is even more striking. The running belt, if it is not central, acquires a lateral inclination always in such a direction as to make it ride up the conical side of the pulley on whichever side it may be, so any tendency to come off is always being corrected, and it rides quietly on the centre of the pulley. With a slipping belt, however, *i.e.*, if it is not running sufficiently fast to counteract the effect of the slipping, the belt moves automatically towards the smaller portion of the cone, and so rides off indifferently on either side, according to the direction in which it happens to start.]

CHAPTER III

PLANIMETERS, ETC.

THE earliest application of the area method for finding the product of force × space is due to Southern, who invented the Watt-Southern steam engine indicator (Fig. 10), which is the parent of all steam engine indicators, such as the "Dark," or the "Crosby" Indicator. In the Southern apparatus a card is moved to and fro at a speed proportional to that of the piston-rod, while ordinates are drawn by a pencil, attached to the end of a small piston-rod, raised against an antagonistic spring, by the pressure of steam in the cylinder. In the original indicator of Watt (Fig. 11) a needle was deflected over a divided dial, and by this means the vacuum produced was found. In Farey's important "Treatise on the Steam Engine," London, 1827, Southern is mentioned: "The calculations which were required for proportioning the dimensions of engines were commonly intrusted to Mr. Southern, who was a skilful mathematician, and to whom Messrs. Boulton and Watt

were induced to give an interest in their manufacturing chiefly on that account."

Recording Apparatus used in Conjunction with Work-measuring Machines.

The earliest recording apparatus employed for this purpose was that of General Morin, 1841, "Notice sur divers Appareils

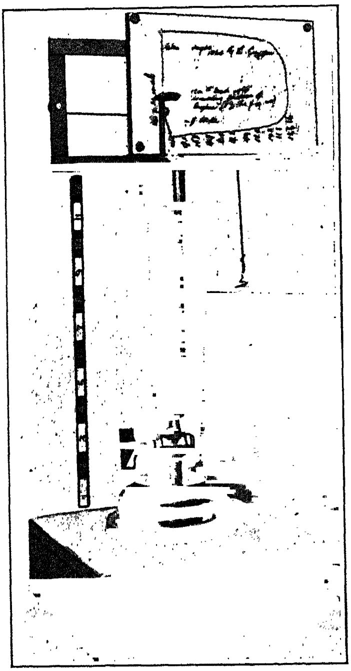


Fig. 10.

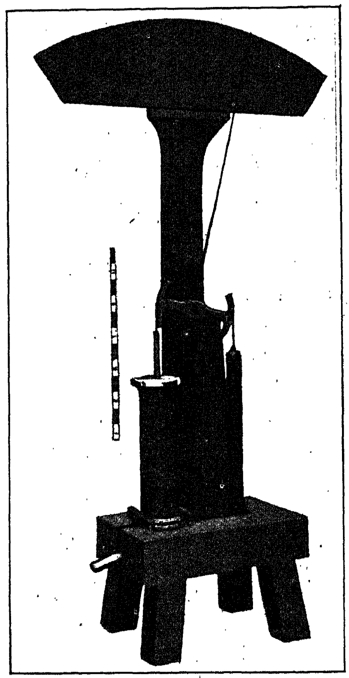


Fig. 11.

Dynamometrique." This apparatus gave the sum of the values of the power transmitted by a dynamometer during any given period. Since the time of Morin many mechanical integrators have been devised for effecting the same summation. Experience, however, has taught the investigator who employs the dynamometer that a knowledge of the *way* in which the power is transmitted is far more instructive than finding at the end of a test the sum of all the elements of the power

transmitted. In the apparatus of Morin a metal disc rotates on an axis, at a rate proportional to space through which the force acts ; a small roller about one-fifth of the diameter of the disc is carried on an axle, parallel to the surface of the disc and cutting its axis. The roller can be slid along the surface of the disc in contact with it, so that its point of contact may be at any distance from the centre of the disc. This distance from the centre is made always proportional to the force acting. It will be seen in what follows that the revolutions of the roller are proportioned to both the space through which the force acts and also to the force and therefore to their product or to *work*. (See note on General Morin.)

In the continuous steam engine indicator of Ashton and Storey the same combination of disc and roller is employed. The radial position of the roller is caused to be proportional to the steam pressure, while the motion of the disc is due to the traverse of the piston ; when the roller is at the centre of the disc it does not rotate, and is at a point corresponding to the atmospheric line of an ordinary indicator diagram. When by virtue of steam pressure the roller is carried from the centre of the disc it is made to rotate with a velocity proportional to the pressure and also proportional to the velocity of motion of the piston of the engine. The principle assumed is that the revolutions of the roller are directly proportional to the work done by the engine in a given time.

In another kind of registering apparatus a band of paper is moved by means of a cylinder or cylinders at a rate proportional to the space through which the force acts, and ordinates proportional to the force acting are continuously drawn by a scribing point. Thus the area generated is proportional to the product of force and the space through which it acts, that is, to *work*.

This method of producing a trace is of great value, since it shows at each instant the amount of force acting. In a dynamometer made from my designs and shown at the Electrical Exhibition in Paris in 1881, the record was made on a paper-covered cylinder, and when the machine was subsequently used to test the power absorbed by a loom the work elements due to the acceleration of each piece of the mechanism in each complete cycle were clearly shown. The trace at once

suggested that careful balancing of the elementary links or parts would reduce the power required to drive the machine. In the experimental ship-model tests as now made by the Admiralty at Haslar, and also by several of the foreign Powers, the diagrammatic registration of power, due to William Froude, is always employed. Recently, in 1907, the same method of recording power has been used in the torsion-meter as applied to ship propulsion by Mr. A. Denny and Mr. Edgecombe. The contrast between the behaviour of the reciprocal steam engine and the turbine was clearly shown by the diagrams, which proved themselves to be very instructive ; in fact before this admirable method of recording the transmission of power was employed, what was really taking place between the engine and the propeller was almost unknown. In Germany good work has been done in the same direction by Frahm. A description of machines for measuring the power absorbed by propellers will be found under the heading "Torsion-Meters. In the figures belonging to the description of torsion-meters facsimiles of original diagrams are shown.

In the work-measuring machines made from the author's designs in 1881–82, in addition to diagrammatic apparatus two mechanical integrators were employed. Fig. 12 shows the construction of a ratchet and link integrator by the author. LK is a link vibrating about the point M. A connecting-rod from the ergometer attached at K moves it to and fro, or it may in practice be driven much faster than the ergometer pulleys by means of intermediate gearing, so that its vibrations are proportional to the revolutions of the pulleys. The T-shaped piece S is controlled in its movement by the spring of the ergometer. Motion in the direction of the arrow would result from the extension of the spring. To this T-piece the piece H is attached by means of a rolling contact R. E is connected by the same device at R′, the piece H is connected to F by a pin which slides in the link L. The piece E works the two arc-shaped pieces CD. These are furnished with pauls acting in opposite directions. The ratchet-wheels are denoted by the inner circles ; the outer ones denote cog-wheels in gear at their point of contact. It is evident that whichever way E moves it will be driving round the wheels AB. These wheels drive an ordinary recording train of wheels. The arc-

ended pieces H and F are used so to connect the systems together that the effect is the same as if connecting-rods of infinite length were used. The ratchet-wheels used in the instrument are small pulleys faced with leather, and the pauls consist of a small bundle of steel wires; by this means at whatever position the paul is it at once engages with the

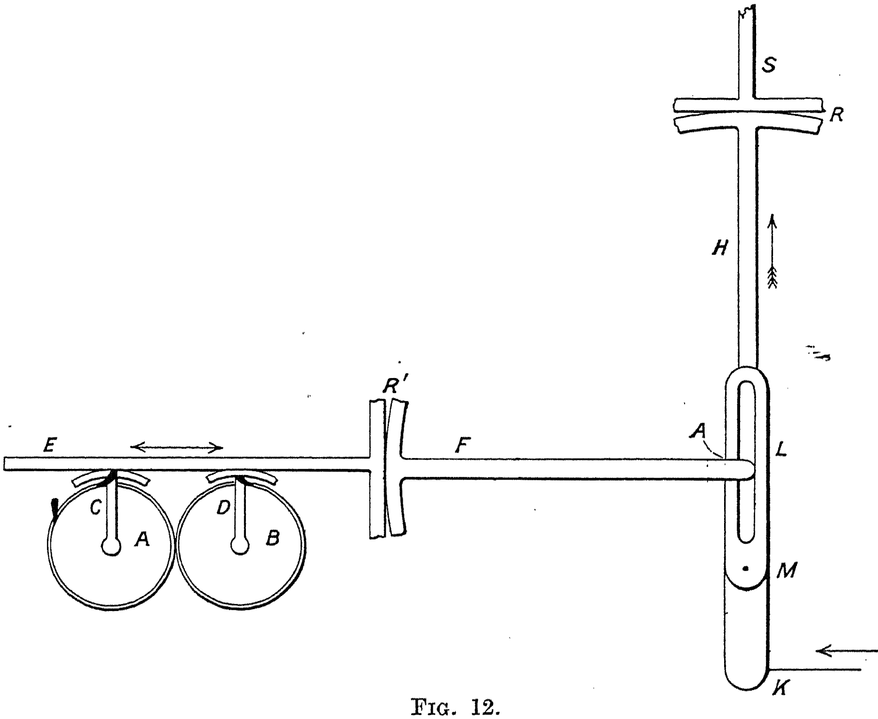


FIG. 12.

wheel to be driven, and consequently there is no loss of time in the action.

In another integrator I used a small cylinder and piston acting as a pump, which raised either water or oil into a measuring vessel. Its *rôle* of action is analogous to that of the integrator just described. The stroke of the pump was made always proportional to the force acting, and the number of strokes to the distance through which the force acted, so that the quantity of liquid delivered in a given time became a

measure of the work done in that time. The little pump was double-acting, so that the flow was practically continuous.

The area of figures in a plane is found thus :—
Let OX, OY (Fig. 13) be rectangular axes, and let any number of straight lines be drawn parallel to OY equidistant from one another, so that the area is divided into a number of elementary areas such as A ; also rectangles may be described each equal to an elementary area A. Then the sum of the areas of all such elementary rectangles equals the area of the whole figure. It is evident that the more numerous the elementary areas,

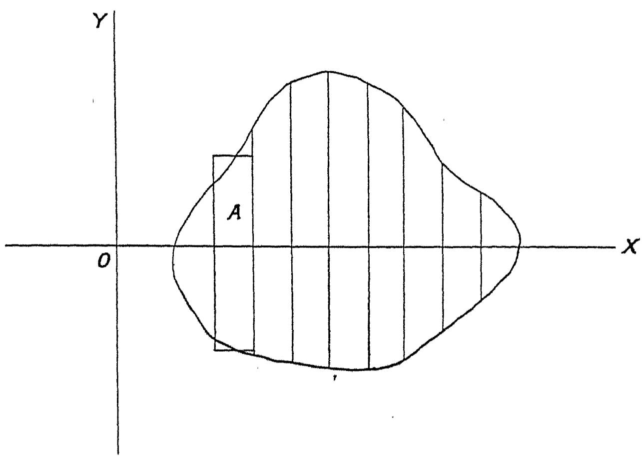


FIG. 13.

the more nearly will their opposite sides equal one another and also the length of the corresponding rectangle.

If the width of an elementary rectangle be Δx, *ab* some distance from O equal to x, and y the length of the elementary rectangle, the area of this rectangle is $y \Delta x$. And the sum of all such quantities equals the area of the whole figure. In the limit when Δx becomes dx, that is, an infinitely small width, the area is represented by

$$\int_a^b y dx,$$

where the integration has been taken between the ascribed limits of x, namely, a and b. If by some apparatus a wheel

is made to rotate at a rate proportional to the value of y and the frame from which it takes its motion traverses OX at a uniform rate, the rotation of the wheel will indicate the whole area contained by the boundary passed over by the wheel.

It would be entirely beyond the scope of this book to give anything more than a sketch of this attractive subject, but sufficient directions will be found for estimating those diagrams which are generated by recording dynamometers of different kinds. Should the reader wish to study the interesting subject of the methods whereby integration has been effected by mechanical means, the following papers and references may be consulted :—

"Amsler's Planimeter": Sir Frederick Bramwell, F.R.S., Report, British Association, 1872, pp. 401—412.

"An Integrating Machine": C. V. Boys, Proceedings of the Physical Society of London, Vol. IV., pp. 199—206; 1881.

On "Integrating and other Apparatus for the Measurement of Mechanical and Electrical forces": C. V. Boys, Proceedings of the Physical Society of London, Vol. V., pp. 8—29; date of paper, November 26, 1881.

"Apparatus for Calculating Efficiency": C. V. Boys, Proceedings of the Physical Society of London, Vol. V., pp. 28—35; date of paper, January 28, 1882.

"Analytical Investigation of the Amsler Planimeter," and also a geometrical treatment of the subject, by Prof. Ball; see "Integral Calculus," B. Williamson, F.R.S., 1884.

"Mechanical Integrators," by Prof. H. S. Hele Shaw, Vol. LXXXII., Proceedings of the Institution of Civil Engineers, 1884—85; p. 92.

"The Hohmann-Coradi Precision-Planimeters": Pamphlet edited by Luckhardt and Alten, Cassel, 1885.

"The Theory of the Planimeter": Prof. A. G. Greenhill, F.R.S., Encyclopædia Britannica, Vol. XXII., p. 721; 1898.

"Integrator applied to Dynamometers," by the author; exhibited Royal Society, 1894. And also description in German: "Deutsche Mathematiker-Vereinigung," Sonder-Abdruck aus dem 1892; "herausgegebenen Katalog Mathematischer Modelle, Apparate und Instrumente."

The Amsler Polar Planimeter as used by Engineers and Ship Builders.

[The Amsler polar planimeter is a neat instrument which has the property of giving directly, by the rotation of a roller called an index-wheel, the area of a plane figure when the tracing point is carried once round the periphery of the figure. It consists of two bars jointed to one another (Fig. 14), while at one end of each there is a point one of which forms a pivot about which the instrument may be turned which carries a small weight, while the other is made to trace the outline of the figure. The third point of support is the edge of the index-wheel, which is carried by the pointer arm and so mounted that the axis of this wheel, the pivot connecting the two arms, and the pointer lie in the same vertical plane. The index-wheel and pivot are often carried on a sliding frame, which can be clamped to the pointer bar at any point and then adjusted accurately by means of a micrometer screw, so that certain marks correspond, when the unit of the vernier reading of the index-wheel will be a simple number of square millimetres or a simple fraction of a square inch or square foot or other unit as indicated by figures engraved on the bar.]

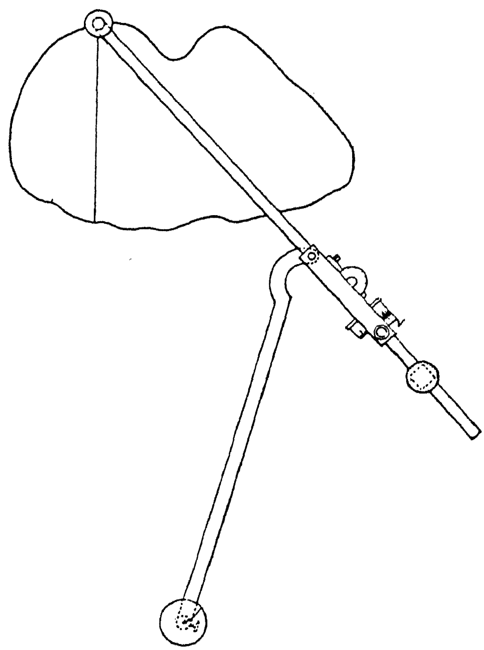


Fig. 14.

The lowest point on the inclined arm (Fig. 14) is the pivot, and the highest is the tracing point. This, which is not sharp, is placed on some mark on the boundary of the diagram which

is to be estimated, and the reading of the index-wheel taken. Then the tracing point is taken round the boundary till the mark is again reached; the index reading is again taken. The difference of the two readings shows the value of the area in terms of the selected unit. In order that the instrument may be calibrated it is convenient to take the tracing point round a known area and note the reading of the index-wheel. This is most accurately done by means of a thin metal radius bar of known length R; one end of this bar is pivoted by means of a projecting needle stuck into the drawing-board, at a point which is roughly the middle point of the area to be found; the tracing point of the planimeter is then placed on a small conical indentation near the free end of the bar so that it is at a known distance R from the pivot. The position of the free end of the bar is marked by a fine line on the paper below it and the index-wheel read; then the radius bar, carrying with it the tracing point of the planimeter, is rotated once and the reading of the index-wheel again taken. The area πr_2 should be shown by the reading of the index-wheel. By very carefully adjusting the position of the slide on the tracing bar of the planimeter the accuracy of the indicating marks engraved on the bar can be tested, or the correct positions determined if error is found. The greatest care should be taken in handling the instrument, as its excellence of performance depends on the joints being in perfect adjustment, that is, free to move without any shake.

The following are the essential points in the construction of the polar planimeter:—

The pin of the joint connecting the two bars, the axis of the wheel, and the end of the tracing point must lie in one plane. The roller or wheel must run easily and free of the vernier. The roller, which is furnished with a tangent screw, drives a counting-wheel once for every ten turns of itself. The primary divisions of the roller are ten in number; these are again subdivided into ten parts, and these are, by means of a vernier, again divided into ten parts, so that one thousandth of a turn corresponding to the one hundredth of a square inch or corresponding unit can be estimated. It is important that the paper on which the diagram to be integrated is drawn is free from ribs; these tend to give the roller a slight rotation when moving at right angles to itself, when there should be no

rotation at all. When a fairly smooth paper is used, practically no error is introduced by its surface.

To use the instrument: set the tracing point to a fine mark on the boundary of the area, and it is well to press the point on to the paper so as to make a slight indentation which can be felt, read the counting-wheel, the roller, and the vernier. Lead the tracing point round in the direction of clock-hands until the mark is again reached, and take the reading. The difference of the two readings gives the area. But two cases have to be considered, the first when the pivot is within the area, and the second when it is outside it. When it is outside, the difference of the two readings has to be taken, but if it is inside the area, then the excess of the second reading over the first gives the area between a circle of known value, called the datum circle, and the boundary of the area. If the figure is less than the datum circle, then the result is negative; therefore the area of the datum circle has to be added to the second reading before subtracting the first from it—what is over equals the area sought.

It is important that the pivoted bar of the planimeter should be so placed that when the tracing point is taken round an area the angle between the bars should not be very great or very small. When it is so, the tracing point is not quite so easily led round the boundary furthest from or nearest to the pivot. The position of the planimeter with respect to the area shown in the figure would be found to work well.

The planimeter when well manipulated gives such valuable results that it will well reward the user to learn its different applications by careful practice. Before planimeters were invented, areas were found by the method devised by Simpson. His rule is exact for parabolic figures of the third degree, but for other figures it only gives the approximate area. The method is interesting and instructive, but requires careful construction and working to obtain good results. Examples of its application are given at pp. 63—66, "Rules and Tables," by W. J. M. Rankine, F.R.S., 1876.

Theory of the Polar Planimeter.

In order that the area or the mean force line may be found from an ergometer diagram, an instrument whereby the area

is found at once by moving a tracing point round its boundary line, is usually employed. The following explanation of the planimeter of Amsler, which is much used in the office of the engineer and shipbuilder, is based on notes taken by the author

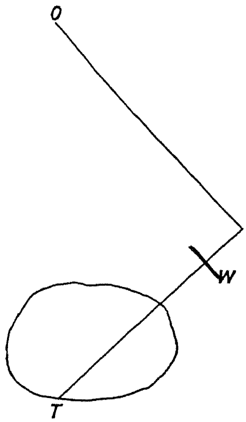


Fig. 15.

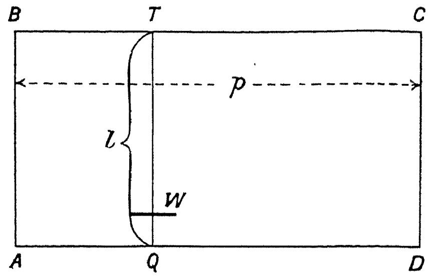


Fig. 16.

at a lecture on the subject given by Prof. O. Henrici, F.R.S., many years ago.

The construction of the Amsler-Laffon * planimeter is shown diagrammatically in Figs. 15—21. The radius bar OQ

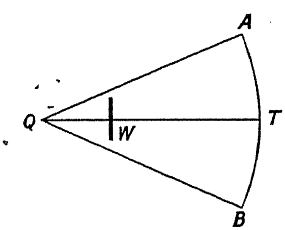


Fig. 17.

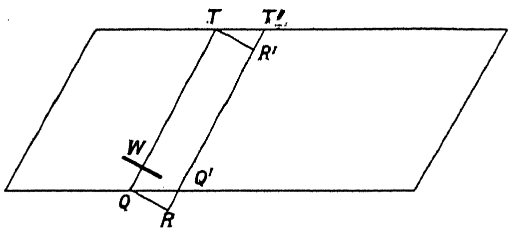


Fig. 18.

and the pole arm QT are hinged at Q ; the bar rests on a horizontal plane, such as paper attached to a drawing board, on three points, viz., the tracing point T, a point of contact of a

* In the year 1856 Prof. Amsler-Laffon invented the polar planimeter called after him. Up to 1885, 12,400 of these instruments were sent out of his works at Schaffhausen : Vol. LXXXII., Proc. I.C.E., p. 14.

small wheel W, and a sharp point O, round which the system is free to rotate while T is moved on the boundary line of any area, the value of which is sought in terms of some unit of square measure, such as the square inch or square centimetre. We may first consider the pole arm of QT and the wheel W apart from OQ. The axis of the wheel is so placed that it lies in a straight line passing through vertical lines through Q and T.

Let TQ $= l$ (Fig. 16), AD $= p$, and let the initial position of TQ be AB ; suppose it to move parallel with itself, from BA to CD, then the area lp is swept out. Now p equals some multiple of the circumference of the wheel W. Calling this w, $p = w$ and the area A $= lw$. Suppose the circumference of W to be divided into n equal parts, each very small and $= u$, then w equals the number of all the u quantities rolled over, and w gives the sum of the elementary areas lu contained in the rectangle. Any unit may be employed by duly arranging the length l and the radius of the wheel W.

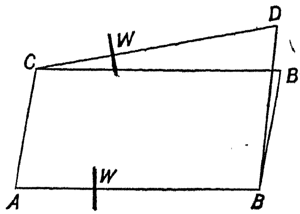


FIG. 19.

Next let the rod (Fig. 17), *i.e.*, the pole arm, rotate about Q ; it will then sweep out an area $= \frac{1}{2} l^2\theta$, where θ is the circular measure of the angle AQB. The wheel rolls over the arc $c\theta$, W being placed c units from Q ; therefore $w = c\theta$, and the area $= \frac{1}{2} \frac{l^2}{c} w$, so that w shows the value of the area swept out.

Next let us suppose the rod (Fig. 18) to move parallel to itself but not in a direction perpendicular to itself. When this is the case the wheel both rolls and slips. Let the rod QT move through a small space to Q′T′ ; this can be resolved into a motion perpendicular to the rod, which would bring the rod to RR′, when the rectangle QTR′R would be generated, and the sliding of the rod along its own axis from RR′ to Q′T′, this second motion not generating an area. In the first movement the wheel's rotation would give QR, and during the second motion the wheel would not rotate. The rotation of the wheel thus gives the value of the area QTT′Q′. Now (Fig. 19) suppose the motion of the rod to be made up of small steps all resolved

in a similar manner, then the rotation of the wheel shows the generated area as before. Again resolving the motion into a large number of small steps let AB move to CD, the step being considered so small that the arcs AC, BD may be taken as straight lines. The area swept out is ACDB ; this motion can be resolved into a step from AB to CB′ parallel to AB, and a rotation about C, from CB′ to CD. Then the rotations of the wheel W when summed up give

$$w = p + c\theta.$$

W is distant, as before, c units from A, and the total area A swept out is

$$\mathrm{A} = lp + \tfrac{1}{2}\, l^2\theta,$$

p being expressed in terms of w.

For finite motion the whole area equals the sum of the areas swept out during the different steps. But since the wheel continuously rolls, the total rotation will be the sum of all the successive elementary rotations. If the whole rotation be called w, if the sum of all the small turnings θ be called α, and the area A, then

$$\mathrm{A} = lw + (\tfrac{1}{2}l^2 - lc)\alpha\,;$$

α is also the angle which the first and last positions of the rod make with one another.

When the planimeter is used, the rod QT is always brought back to its initial position, so that $\alpha = 0$, or should the rod be turned once entirely round $\alpha = 2\pi$.

In the first case $\mathrm{A} = lw$

In the second case $\mathrm{A} = lw + (\tfrac{1}{2}l^2 - lc)\ 2\pi$,

when the rod is turned round once.

Since the point Q in the first case only moves to and fro on an arc of a circle, no area is generated by it. But in the second case, if the pole O be situated within the area, the rod QT describes the area between the boundary of the figure and the circle having a radius r equal to OQ, and the rod turning once round makes $\alpha = 2\pi$, so that in this case the total area A

$$\mathrm{A} = lw + (\tfrac{1}{2}l^2 - lc)\ 2\pi + \pi r^2$$

or

$$= lw + \mathrm{C} \text{ where } \mathrm{C} = (\tfrac{1}{2}l^2 - lc)\ 2\pi + \pi r^2,$$

a constant depending on the dimensions of the instrument, which will be found marked on one of the bars. Where several standard positions are marked on the tracing bar representing

different units, corresponding constant values will be found given for each.

[It may be well to add that where the index-wheel is placed beyond the hinge Q, as is usual in instruments in which the hinge and wheel are carried on a sleeve itself capable of sliding on the bar TQ, so as to read square feet, square inches or square millimetres or other unit according to the mark to which it is set (Fig. 14), the term $-lc$ in the preceding equations should be $+lc$. It will be evident that if the point T is carried round the pole O in a circle of such size that the wheel does not roll at all, the plane of the wheel will pass through the pole.

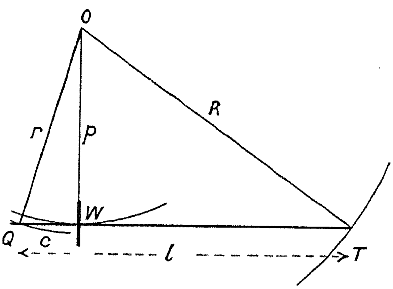


FIG. 20.

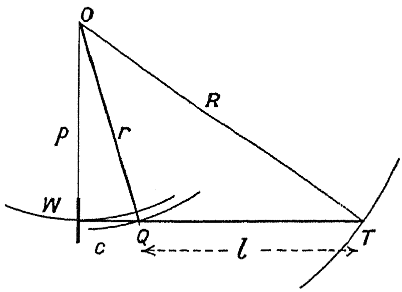


FIG. 21.

An inspection of Figs. 20 and 21 will then show that p, the perpendicular distance from the pole to the wheel, r, the length of the pole arm, and R are the radii of three circles described in this movement; also that

$$\begin{aligned} p^2 &= r^2 - c^2 \\ R^2 &= p^2 + (l \pm c)^2 \\ &= r^2 \pm 2cl + l^2, \end{aligned}$$

and the area of the circle of radius R described by T $= \pi(r^2 \pm 2cl + l^2)$, which is the same as is given above. Of this πr^2 is the area of the circle described by the joint Q while $\pi l^2 \pm 2\pi cl$ is the area swept by the radius rod l outside the circle described by Q. That part of the radius rod which is within this circle in Fig. 20 sweeps over it twice, once positively and once negatively, and this does not count.]

Mechanical Integrator.

Mechanical Integrator used in connection with the Ergometer.

A metal cylinder AB is carried on an axle CD (Fig. 22) which rotates in a frame suspended from a steel rod, which moves in two V-grooved pullies (see Fig. 23 and picture from photograph). The frame is connected to a double-grooved semicircular disc Q, which forms part of the frame FH which carries the

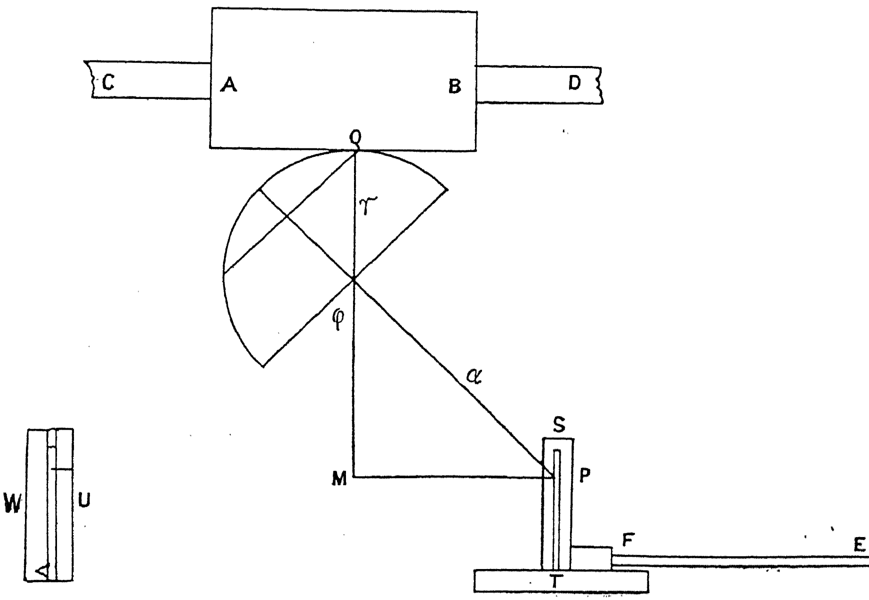


Fig. 22.

hemisphere, by two cords; this keeps the cylinder always in rolling contact with the hemisphere when turned through an angle about its vertical axis; a counter attached to the axle of the cylinder shows the number of revolutions it makes, and hence, as will be shown, the work transmitted by the machine.

The horizontal motion is given to the hemisphere through the arm carrying the stud E (Figs. 23 and 24) and frame LH which is attached to the arm, a continuous cord passing round a V-groove on a great circle on the hemisphere, and through its vertical axis rotates the hemisphere. Thus the cylinder and

hemisphere are in perfect rolling contact when the hemisphere is moving about either axis, separately, or about both axes at the same time. The stud P (Fig. 22) moves in a slot ST,

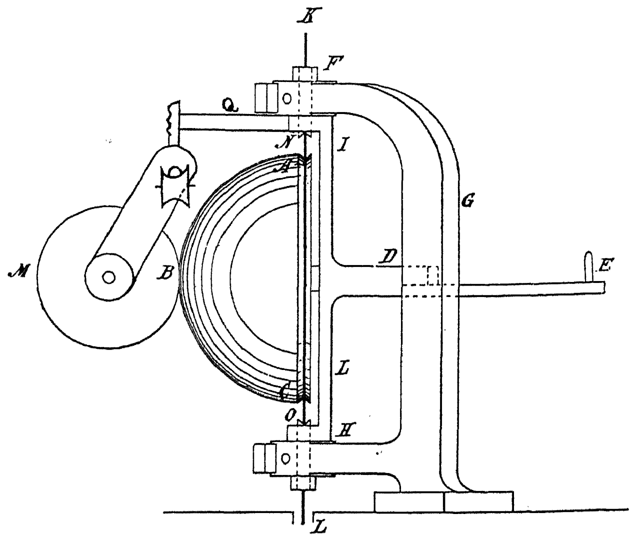


FIG. 23.

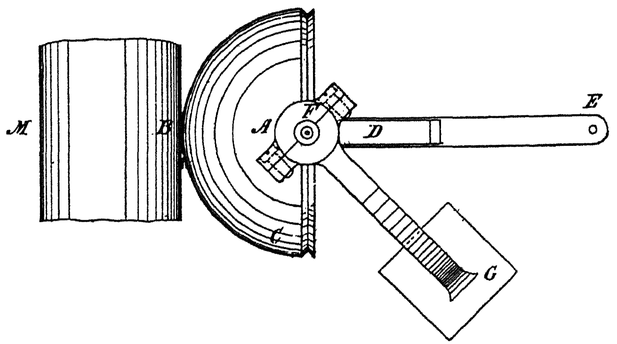


FIG. 24.

carried on a geometric slide having five surfaces of contact, shown in a side view, WV. The length MP (Fig. 22) is proportional to a force when the instrument is used in connection with ergometers. The theory of the instrument is as follows :—

If the rod FE (Fig. 22) is pushed through a distance y, then

the hemisphere moves through an angle φ about its vertical axis, so that $a \sin \varphi = y$, where a is the distance from the centre of the hemisphere to the stud P. Since the hemisphere and cylinder are in contact, and $c =$ radius of cylinder, and $R = r \sin \phi = \frac{r}{a} y$, and $r =$ radius of hemisphere, we have $c d\theta = R d\psi$, where $d\psi$ is the angle turned through by the hemisphere on its horizontal axis, and $d\theta$ the angle turned through by the cylinder so that we get

$$d\theta = \frac{r}{ac} y d\psi$$

Since the revolution of the hemisphere is caused by the cord which engages in the V-groove in the hemisphere, the angle $d\psi$ is proportional to the motion dx through which a point on the cord has moved, so that $d\theta = A y dx$,

$$\text{and} \int y dx = B\theta,$$

where A and B are constants depending on the dimensions of the instrument. θ is measured by means of a dial on the axis of the cylinder.

The instrument is used in connection with a work-measuring machine shown in the photograph, and it is so mechanically arranged that x is proportional to the space, and y to the force acting through the space x, so that $\int y dx$ gives the work transmitted by the ergometer.

For many purposes the mechanical integrator is of considerable value, but when fluctuations in the rate of transmitting work take place, then the best method for obtaining the value of the work transmitted is from a diagram produced automatically on a cylinder carrying paper, in which the traverse of the paper is proportional to the distance through which the force acts, and the ordinate is proportional to the force at any instant. The area then gives the value of the work transmitted. In testing machines in which periodic motion exists, such a diagram shows continuously *how* the work is transmitted. In one form of my ergometers used in testing spinning machinery (at Messrs. L. Crossley's, Halifax) the diagram method is always used, as it shows at what rate any particular part of the machine is absorbing energy. The different moving parts

are connected electrically with electromagnetic styli which mark the diagram, so that the behaviour of each part is clearly indicated.

The torsional form of ergometer has been in continuous use for the last two years in the Millard Mechanical Laboratory, in connection with the experimental determination of the mechanical equivalent of heat from the rotation of copper cylinders in the magnetic field.

Boys's Engine Power Meter.

[Boys's Engine Power Meter was mentioned in the table of contents prepared by the author, but he had not written any description of it. This instrument depends on the principle employed independently by Abdank Abakanovicz, and C. V. Boys in their integraphs. If while a point P is made to trace any given curve AA (Fig. 25) a steering wheel W is by some means always kept vertically above the point P and parallel to the line PQ, the base QM being maintained constant, then the wheel W will trace out a curve such that the tangent of its inclination is numerically equal to the height PM if the base QM is unity. The curve BB' therefore gains in height in proportion as the area between the curve AA and the axis of x gains

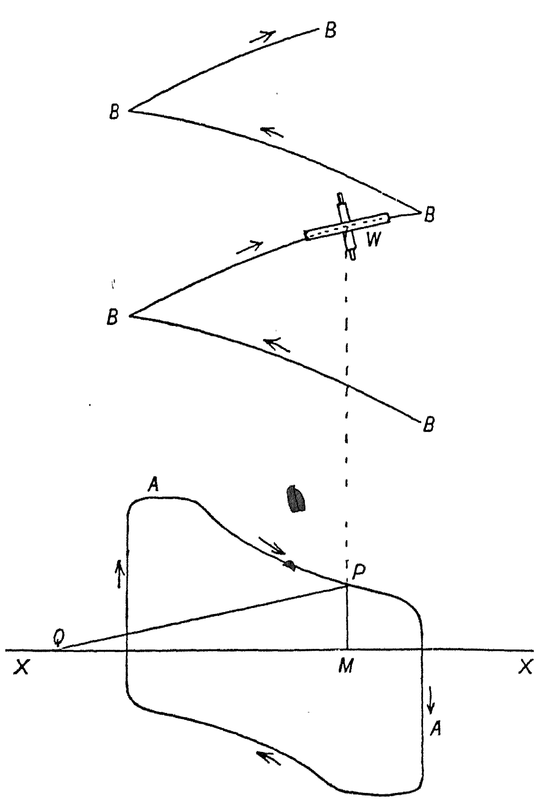


Fig. 25.

in area, and the process is a mechanical realisation of mathematical integration. The curve AA in Fig. 25 represents the combined indicator diagrams of a steam engine taken at the two ends of a cylinder and the curve BB traced by the steering wheel W is the integral curve. It will be seen that on the return journey from right to left the slope of the line QP is reversed in direction also, and the steering wheel W having its direction of motion and slope both reversed, continues to mount, thus adding the area below the line xx to that above the line. The height BB of any one zig zag $\times$ QM = area enclosed by the curve AA. In order to make this construction practicable as a steam power integrator an ordinary indicator cylinder is used with the two

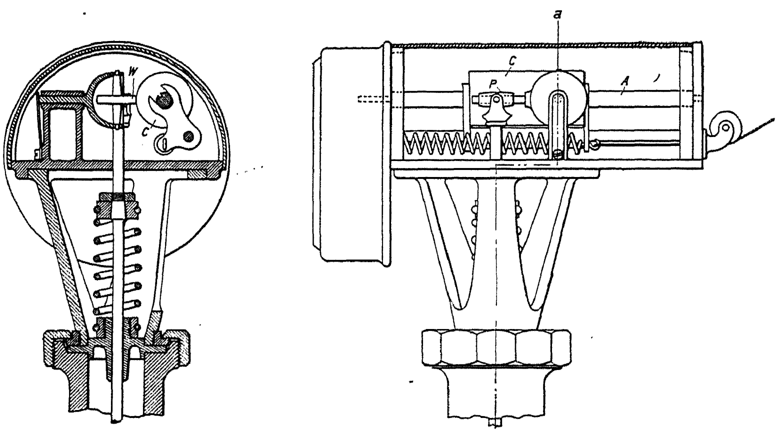


FIG. 26.

ends put into communication with the two ends of the steam cylinder of the engine. In the place of one double-acting instrument two single-acting instruments may of course be used one connected to each end of the steam engine cylinder. Considering now the double-acting instrument, the upper end of the cylinder is seen in Fig. 26 of which the left hand part is a vertical cross section through the broken line *aa*, while the right hand part is a side elevation with the cover only in section. The piston rod of the indicator is seen with the indicator spring above the cylinder connected to the piston-rod, so that the displacement of this above or below its neutral position is propor-

tional to the excess of pressure on one side of the piston over that on the other. The upper end P of the piston-rod is in the form of a swivel sleeve engaging a rod, which projects radially from a bell within which the wheel W is free to rotate, and the bell with its wheel W is turned more or less by movement of the piston-rod, so that the tangent of the inclination of the rod and of the wheel W is proportional to the displacement of the piston-rod. The bell is pressed by the action of a spring towards a light drum C against which the wheel W is pressed, and this drum is given a reciprocating movement in time with and proportional to the motion of the piston of the engine by means of a flexible connection acting against a spring in a manner made clear in the figure. The hemicylindrical case containing the integrating mechanism can be turned upon the cylinder so that the flexible connection points in the desired direction and it may be locked in that position by means of a union nut. When the integrator is working the wheel W is inclined in accordance with the steam pressure, while the drum C is drawn under it in conformity with the motion of the piston of the engine. The wheel is unable to move vertically as in the integraph, but the surface of the drum is free to move under the wheel instead, and so it rotates at any moment at a rate proportional to the product of the effective pressure multiplied by the speed of movement of the engine piston, *i.e.*, to the rate at which work is being performed by the steam on the engine piston, and the whole rotation of the drum C transferred by the axle A, on which it slides with a feather connection, to a counter in the box at the left-hand end gives on a set of dials there the integrated indicated work of the engine over any length of time. A hemicylindrical cover springs on and protects the integrating mechanism. Unlike the power meter of Ashton and Storey, there is no sliding between the integrating surfaces of this integrator. If the spring is not the exact length needed to bring the wheel W into its neutral position when the piston is unacted on by steam pressure, the record due to any complete number of strokes is not affected, as the tangents of the inclinations of the wheel W will be increased during alternate strokes to the same extent that they are diminished during the intermediate strokes. It will be seen that if

D is the diameter of the engine cylinder } in the same
d ,, diameter of the indicator cylinder } units
L ,, stroke of the engine piston } in the same units
l ,, stroke of the integrator drum }
S ,, stiffness of the spring, *i.e.*, ten times the number of pounds required to move it one-tenth of an inch
K is the distance in inches between the axis of the piston-rod and the centre of the wheel W
r ,, radius of the drum C in inches
n ,, number of turns of the shaft A recorded in the counter
N ,, number of foot-pounds of indicated work

then $$\mathrm{N} = \frac{\mathrm{D}^2}{d^2} \times \frac{\mathrm{L}}{l} \times \mathrm{SK} \times \frac{\pi n r}{6}.$$

The coefficient of n may be determined once for all ; calling this k, then $\mathrm{N} = kn$.]

As an example of the use of a planimeter in dynamometry the reduced trace (Fig. 27) is given. This is a dynamometer record taken during an experiment on a model ship drawn through water in a testing tank. The traces showing the large oscillations exhibit the force required for towing the model ship. The traces which are rather darker, showing smaller oscillations, exhibit the thrust of the propeller. The top broken line shows the revolutions per minute of the screw. The middle broken trace shows the distance traversed, viz., 25 feet, between each break, also the lowest broken line shows the revolutions per minute of a screw. The horizontal lines ruled through the traces show different values of the mean forces involved.

For example, in a diagram taken from a model ship-testing ergometer the following numerical values were obtained :—

The force = 3·5 pounds, and it acted through 25 feet, so the work done = 3·5 × 25 = 87·5 foot-pounds. The space 25 feet was represented by 1·83 inches on the diagram,* while the load of 3·5 pounds was represented by 8·9 inches so that an area of the diagram of 8·9 × 3·5 = 16·189 square inches represented 87·5 foot-pounds work done, and one foot-pound of *work* was represented by $\frac{16 \cdot 189}{87 \cdot 5} = 0 \cdot 18512$ square inch.

* The figure is reduced from the original diagram.

Periodic Curve and Record.

In many ergometer tests the diagram shows a well-marked periodic curve. This is very evident when the force acting is

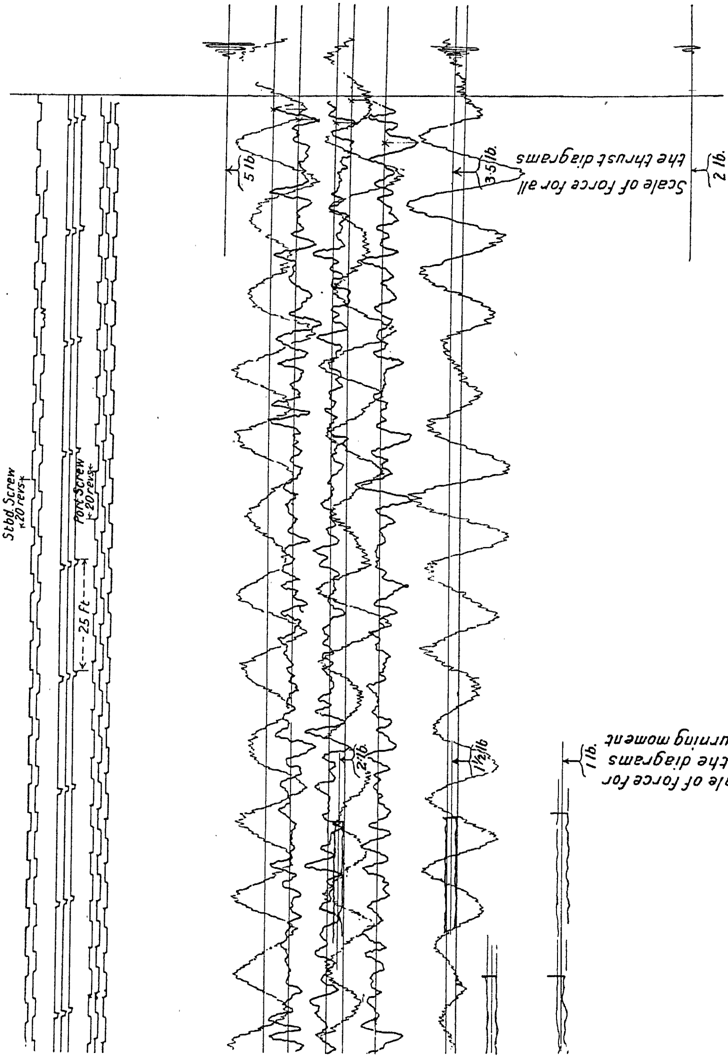


FIG. 27.

communicated through an elastic link, such as a steel towing-rope. It may also arise from other causes. In order to deal

with a diagram of this kind, a mean line of force is found and at once multiplied by the space through which the force has acted. The operation for finding this mean line is that of integrating the area of the diagram between convenient limits and dividing the area so found by the length of the base line. Or another method is to draw a mean line by eye, cutting each wave of the curve, and then integrating with a planimeter, such as that of Amsler or Coradi, the whole surface bounded by the curve, half of which lies roughly above the assumed mean line and half below it. If after traversing a length of the curve containing an equal number of crests and hollows and returning by the assumed mean value line to the starting point, the reading of the planimeter is zero, this line has been correctly drawn, but if some small area is indicated, then the line must be shifted through the distance deduced from the reading ; it will then become the *mean value of ordinate* line.

In certain cases, such as the towing of a vessel, through an ergomometric apparatus, the band of paper used for recording the force ordinates cannot be driven at a rate directly proportional to the space through which the force acts, but is driven continuously forward by means of clock-work regulated by a centrifugal governor of the type in which a spring control takes the place of gravity. But it must be noticed that the record so made is of *impulse*, F*dt*, not of *work*, F*ds*. If the quantity, F*dt*, be divided by the whole time that F acts, the quotient will give the mean value of the force F, but if under this condition *work* is to be estimated, then as the ship passes known distances, determined by shore marks, the position must be marked on the diagram at these points at each instant they are passed and the mean force estimated between them. Then by multiplying this mean force by the distance between the marks an approximate value of the *work* during the interval is obtained.

CHAPTER IV

FRICTION BRAKES

SOME APPLICATIONS OF COILED ROPES.

IN very remote times it was well known that a spindle might be rotated by means of a thong or rope coiled about it. In the adventure of Ulysses with the Cyclops (Homer, "Odyssey," IX.), Ulysses, assisted by his companions, when in the cave

of the Cyclops bored out his eye with a bar of olive wood, "having bound it with a thong on each side, they move it, and it constantly runs round." Ulysses after chaffing the blinded Polyphemus, the Cyclops, in polite and polished Greek, escapes from the grasp of the monster while clinging to a fat ram going out to pasture as the rosy-fingered morn dawned. In the earliest lathe the material held between points was rotated to and fro by means of the cord of a bow coiled at least once round it. It was so in the time of Virgil, and it was probably used in the same manner ages before his time.

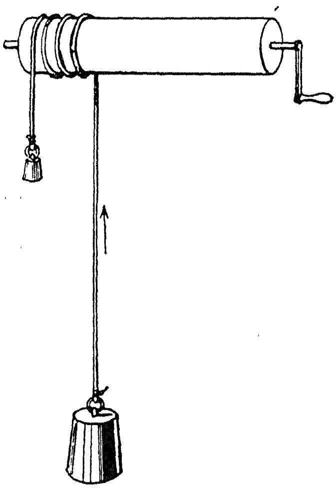


FIG. 28.

The use of the coiled rope must have been known to sailors long ago, who moored their vessels to trees or short posts projecting from the wharf. Such posts have led up to the iron bollard, in some cases of enormous size, so that a steel rope may be coiled round it. If a flexible cord be coiled round a cylinder as shown in Fig. 28, and weights be attached to its ends, the friction of the cord increases greatly with the angle embraced by it. Suppose that the cord embraces half the circumference, and the relative value of the two weights be such that the greater weight is balanced by the lesser one plus the friction of the cord on the cylinder. If we assume that the mean value of friction as about one-third, or more nearly ·35, of the pressure which causes it, then any weight tied to one end will support a weight at the other end three times as great.

Coils.	Weight.	Coils.	Weight.
0·5	3	2·5	243
1	9	3	729
1·5	27	3·5	2,187
2	81	4	6,561

When another coil is taken round the cylinder the small weight will carry one twenty-seven times as great, and each coil multiplies the friction roughly nine times, and also the half-coils three times. It will be noted that half a coil necessarily forms part of the sum of all the coils.

If the small weight be raised a little so as to release the half-coil slightly from the cylinder, the greater weight will descend, but if the small weight be again allowed to act it will be brought to rest. This is true, too, when a ship is warped to a bollard; a slight release of the slack end of the rope enables one man to control with ease the paying out of a rope, which has a great pull on it due to a ship in moving water. This property of the coils of a rope is utilised in such devices as hand-gins for raising or lowering builders' materials, and also in raising and lowering buckets alternately into a well. In this case the empty bucket corresponds to the small weight and the full one the large weight previously mentioned. This way of using the coiled rope is inconvenient when the well is deep, since the lateral movement of the rope as it coils on to the cylinder is limited by the width of the well. An invention of Sir Christopher Wren is so excellent that I give a *verbatim* reproduction of it from the Royal Society, May 5, 1670 :—

"Having considered, that the ways hitherto used in all Engins for winding up Weights by Roaps have been but two, Viz. the fixing one end of a Roap upon a cylinder or Barril, and so winding up the whole coyle of roap; the other by having a Chain or a loose roap catching on teeth, as is usual in clocks; but finding with all that both these wayes were inconvenient the first, because of the riding of much roap in winding one turn upon another; the other, because of the wearing out of the Chain or roap upon the teeth, I have, to prevent both these inconveniences, devised another, to make the weight and its counterpoyse bind on the cylinder, which it will doe if it be wound three times about. But because it will then in turning, scrue on like a worm, and will need a Cylinder of very great length, therefore if there be two cylinders, each turned with three notches and the notches be placed alternately, the convex edges to the concave as in the figure adjoyned, the roap being wound three times about both cylinders, will bind firmly without slyding and working up the weight with a proportionable counterpoyse at the other end of the Roap."

The original picture (Fig. 29) in Wren's paper is not very

clear, so I show a reproduction of it from "Principles of Mechanism," 1870, by R. Willis, M.A., F.R.S., Jacksonian Professor of Natural and Experimental Philosophy in the University of Cambridge, by the kind permission of Longmans, Green & Co. (Figs. 30 and 31) :—

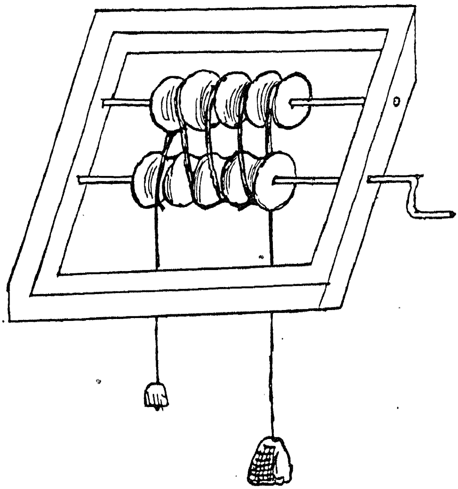


FIG. 29.

"The course of the cord is over the back roller from A to B, under the rollers at BC, and over them at CD, and so on to E,F,G, over at GH, but after passing under H the end of the cord is taken, not under, but over at I, whence the end is allowed to hang down vertically, and has a small weight tied to it of sufficient magnitude to keep the cord in contact with the surface of the notch."

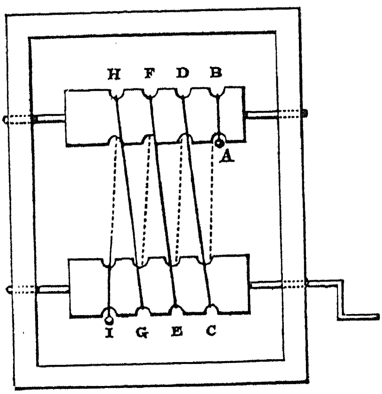


FIG. 30.

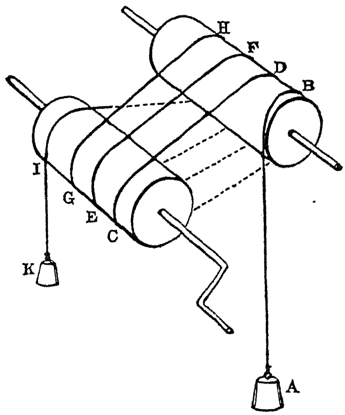


FIG. 31.

By means of this excellent combination of the grooved cylinders the ends of the cord which carry the load and the

counterpoise hang in vertical lines when the machine is at work and the frictional resistance due to several coils is effectual. The sum of the frictional resistances of the rope in passing round the two halves of the *two* cylinders is the same as if it was coiled the same number of times round *one* cylinder, so that both the increase of diameter due to successive coils and the lateral motion of the hanging ropes is entirely avoided. This excellent device, as an invention of Wren, appears to have been lost sight of, and reinvented by several persons, M. Boulogne in 1702, J. Bernouilli and Ludot in 1741, and in 1805 the Society of Arts rewarded Boswell for a similar invention. The same device has been applied to produce reciprocation of motion as required in a mangle. It will be seen in what follows that it is on the property of the coil friction of ropes that the excellence of a certain class of brakes depends.

This method of coiling the same rope round several cylinders has also been turned to good account in one form of the rope brake of William Thomson (Lord Kelvin).

The Rope Dynamometer Brake.

In this type of brake a rope is coiled once round a pulley or flywheel driven by the engine to be tested. A weight W is suspended from its lower end; and its upper end is attached to a spring balance of the Salter type. The direction of rotation is such that the rim moves upwards on the side where the ends of the rope are situated. The spring balance can be so adjusted that the rope may hang in a vertical line when touching the wheel of the engine. For all practical purposes this is a sufficiently good approximation to the real physical condition of a rope in contact with a curved surface. Owing to the fact that the rope is not really perfectly flexible, it cannot hang in a direction truly tangential to the curved surface, so that there is a tendency inherent in the rope to make the efficient radius at which the forces act slightly larger than it is usually assumed to be. In the case of a small pulley embraced by a rope of considerable diameter this increase could not rightly be neglected. If instead of a *quasi*-flexible rope a chain such as those used in the cycle be used, each link being faced with a suitable rubbing surface, a condition of

brake could be set up which is almost absolutely free from the minute source of error introduced by the employment of a rope; but this increase of accuracy appears in practice to be more than balanced by other troubles introduced by the employment of a steel chain with faced links. When more than one rope is employed they are kept in place by distance-blocks of wood arranged to prevent side slip No metal, if used to attach the blocks to the ropes, should come in contact with the face of the wheel, since such contact produces great and dangerous heating. If R is the distance between the perpendicular let fall through the centre of the wheel and a parallel line through the centre of gravity of the weight, we have two reacting moments, namely, the force of friction f acting at the radius r of the wheel and the effective weight $(W - p)$ acting at the distance R. p is the pull shown on the Salter's balance, when the wheel is rotating at the required speed of N revolutions per minute and

$$fr = (W - p)\,R$$

$$\text{the B.H.P.} = \frac{2\pi\,(W - p)\,RN}{33{,}000}$$

$\pi = 3{\cdot}14159$ and 33,000 is Watt's Horse Power constant. If μ (the coefficient of friction) changes, then W changes its position until the reaction becomes steady. It has been already shown that $\frac{W}{p} = e^{\mu\theta}$, where θ is the angle in radians embraced by the rope and $e = 2{\cdot}71828$. In the case described, since the rope entirely embraces the wheel, $\theta = 2\pi$.

As θ increases $\frac{W}{p}$ increases quickly, so that if θ is large p may be a small fraction of W, and consequently the small errors in estimating the readings of the spring balance are not of great importance.

The governing power of this brake is excellent, and it can be easily applied and used in estimating the brake horse-power of many kinds of motors or prime-movers. The brake used as a dynamometer is thus described by William Thomson in his patent No. 437, A.D. 1858.

"Part Fourth. I use the arrangement which has been described for the purpose of testing the action of water wheels, steam engines, and other prime-movers. In performing this part of my invention I employ

the prime-mover to drive a rotating body resisted by the friction of a band, which is held at the end or part where the tension is greatest by a regulated force, and at the end or part where the tension is least by a spring balance or other indicator of force. The difference between the regulated force and the force indicated by the spring balance or otherwise is the resistance overcome at the rubbing surface of the rotating body, which, being multiplied by the velocity of that surface, gives the work performed in a given time."

When a very thin friction band is employed, the difference between r and R for a wheel of considerable diameter is so small that it may be neglected.

The Rope Dynamometer Brake of William Thomson (Lord Kelvin).

The patent specification of William Thomson, No. 437, 1858, for improvements in apparatus for applying and measuring resistance to the motion of rotating wheels, shafts, or other rotating bodies, is so full of valuable matter relating to brakes that I give here a brief analysis of it.

In the experiments made by the Society of Arts, 1888—89, the Thomson brake was employed, but its inventor appears to have been forgotten. Probably the apparatus would now be better known if some diagram had been added to the specification, which is tersely written, and more academic in its wording, than is usual in patent specifications of more recent date. The matter contained in the specification may be described thus. A flexible band is wrapped round any part of the circumference of the pulley which is driven by a prime-mover, and it may be wrapped round it one or more times. To one end of the band a regulated force, *i.e.* a weight, is attached, which opposes the motion of rotation, and to the other end a spring balance is attached. The adjustable weight is applied where the tension is greatest, and the tension at the other end is left to vary with the change of friction, so that by this arrangement the resistance cannot exceed the amount of the force due to the adjustable weight. If the adjustable force due to the weight were applied to the end where the tension is least the resistance might increase beyond that of the adjustable force, and irregular motion would result, or the sudden stoppage of the rotating body.

The band is in some cases wrapped round the whole or any portions of the circumferences of a number of different pulleys, moving on parallel axes, geared together or not, or it may be wrapped round one of these pulleys more than once, one end being acted on by the adjustable force before mentioned, and the other end is either fixed or attached to a spring balance, indicating the tension. This arrangement is designed to resist the motion of different pulleys to an amount which may be modified by regulating the forces at each end of the band.

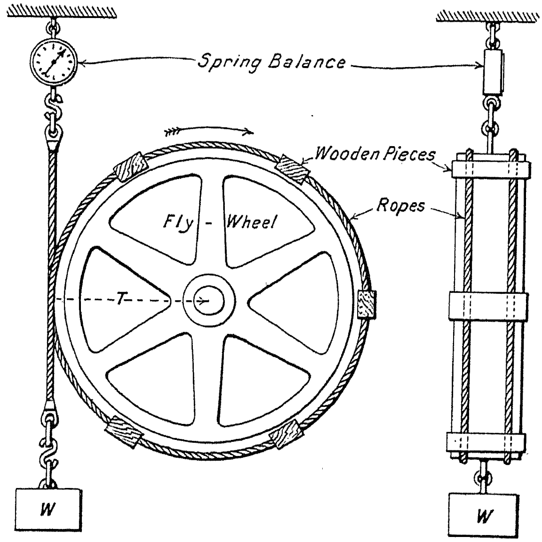


FIG. 32.

The brake described was to be employed in setting up resistance opposed to the egress of a cable. In order that the rubbing surfaces might be renewed without stopping the pulleys, a band is used longer than that absolutely required, and this is gradually paid out so that new surfaces may be successively exposed to friction, also new surfaces of the pulleys may be exposed to friction.

The invention is also designed to test the action of water wheels, steam engines and other prime-movers. The prime-mover is employed to drive the pulley, the motion of which is resisted by a rope coiled on it, one end where the tension is

greatest being subject to an adjustable force (due to a suspended weight), and the other end, where the tension is least, to a spring balance or force indicator (Fig. 32). The difference between the force due to the weight and the force indicated by the spring balance is the resistance overcome at the rubbing surface of the rotating body, and this multiplied by the velocity of the surface gives the work performed in a given time.

In each case of the application of the friction band or rope to a pulley springs may be attached at different points, and so adjusted that when no tension is applied to the band, it is drawn out of contact with the rotating pulley. There are five claims which embody the above subject-matter. Probably the primary object of the patent was to protect an invention important in laying telegraph cables, and from it the ergometer brake very naturally followed.

Prof. W. E. Dalby informs me that Lord Kelvin told him that he invented the brake primarily in connection with the laying of the Atlantic cable for braking the cable as it was paid out.

FLEXIBLE BAND DYNAMOMETER (by W. C. Unwin, from a paper read before Section G of the British Association, 1883).

The ordinary strap brake dynamometer, in which two weights are suspended from a strap which embraces a pulley, driven by the prime-mover to be tested, is described. If the suspended weights be called P and Q, of which Q is greater than P, then the work consumed in friction is expressed by $v(\mathrm{Q}-\mathrm{P})$, where v is the surface velocity of the pulley, or otherwise if θ be the arc embraced by the belt, and μ equals the coefficient of friction, then $\frac{\mathrm{Q}}{\mathrm{P}} = e^{\mu\theta}$, where e is the base of the hyperbolic logarithm, or for a given arc of contact $\mathrm{Q} = k\mathrm{P}$, k depending on the coefficient of friction. For the weights to remain at rest the coefficient of friction must be exactly constant. But since such a condition cannot be attained, oscillations are set up, which prevent very close readings. Such a condition has been modified in the Ayrton and Perry machine. If in place of one of the weights a spring balance be

employed, the dynamometer automatically adjusts itself to changes in the coefficient of friction. The author of the paper goes on to show how the friction of the pulley may be made still more independent of changes in the coefficient of friction.

The method is as follows: Two pulleys, grooved for convenience, are on the driven shaft, side by side. The band is attached to a spring balance placed below the pulleys; it then embraces one of the pulleys, then passes downwards, and embraces one pulley and returns upwards to the second driven pulley, which it also embraces; from its free end a weight is suspended. By means of this arrangement an alteration of 20 per cent. in friction will alter the quantity Q − P less than 6 per cent. Again, if the band be taken over four pulleys, then a variation of 20 per cent. in the frictional coefficient would alter the friction on the pulleys $1\frac{1}{4}$ per cent. By means of this four-pulley brake 3 feet in diameter running with a surface velocity of 50 feet per second a flexible wire band, carrying 100 lbs. as the greater load, 8·8 horse-power could be absorbed. Since two or three wires might be used side by side, each carrying 100 lbs., large amounts of horse-power could be conveniently absorbed, the grooves and bands being cooled by jets of cold water.

Application of the Rope Brake.

THE ROPE BRAKE, USED IN TRIALS OF MOTORS AND ENGINES BY THE SOCIETY OF ARTS, 1888—89.

These exhaustive tests of the output of different kinds of engines, including the

Atkinson Gas Engine . . .	B.H.P.	9·48	Duration of run, 6 hours
Crossley Gas Engine . . .	,,	14·74	
Griffin Gas Engine	,,	12·51	
Paxman Portable Steam Engine.	,,	19·44	

were made by the late Prof. John Hopkinson, F.R.S., Mr. Beauchamp Tower, and Prof. Alexander Kennedy, F.R.S., who were appointed by the council of the Society of Arts as judges of the engine trials. The brake horse-power (B.H.P.) was in every case determined by means of a rope brake on the flywheel or on the flywheels of the engines under examination.

I have found a few engineers who speak disparagingly of the rope brake, but I have also found that their opinions were not based on personal experience of the brake; they certainly would have been satisfied with it had they used it. In the hands of W. Thomson (afterwards Lord Kelvin) it was found to be a satisfactory brake, and a workable form of the brake was patented by him in 1858, as previously mentioned. In the engine trial I have cited, excellent results were obtained from its employment. In the report of the judges for the Society of Arts, 1880, we read, p. 5 :—

"The brake horse-power was in all cases ascertained by means of a rope brake upon the flywheel, or flywheels, of the engines. Two ropes were used for each wheel; they were kept at a proper distance apart and in fixed position upon the flywheel by means of transverse wooden distance pieces. The dead load was applied by means of weights, and the back tension necessary to put the friction on the brake by means of a spring balance. The spring balance was read every five minutes, and its tension was deducted from the dead load applied. This brake was found to work perfectly satisfactorily, and its results are certainly beyond suspicion. It is important, however, if any metal be used for attaching the wood cross-pieces to the ropes, that it shall not rub against the rim of the flywheel; if this should occur, the metal becomes exceedingly hot, and is liable to burn the rope."

The duration of an engine test in one case lasted 6·43 hours, the brake horse-power being 18·95, the weight on the lower end of the rope was 320 lbs., the spring balance reading 32 lbs.: so that the net brake load was 288 lbs. The flywheel in this case was of trough section, and water dripped into it and evaporated. No lubricant of any kind was employed on the surface of the brake. Manilla rope has usually been employed in this type of machine, and has given excellent results.

In Fig. 33 a modification of the Thomson brake is shown. In place of the large weight acting by gravity, a Denison steelyard gravity balance was employed. The rope embraced the lower half of the pulley, a spring balance being attached to the other end of the rope. This form of brake was used in testing machines in the works of Messrs. Willans and Robinson. The figure is reproduced from an excellent paper in the *Engineering Magazine* of November, 1904, by Capt. H. Riall Sankey and Mr. C. Humphery Wingfield. I am indebted to the manager

of this magazine for his kind permission to reproduce the cut. In engine tests made by the authors of this paper the hydraulic

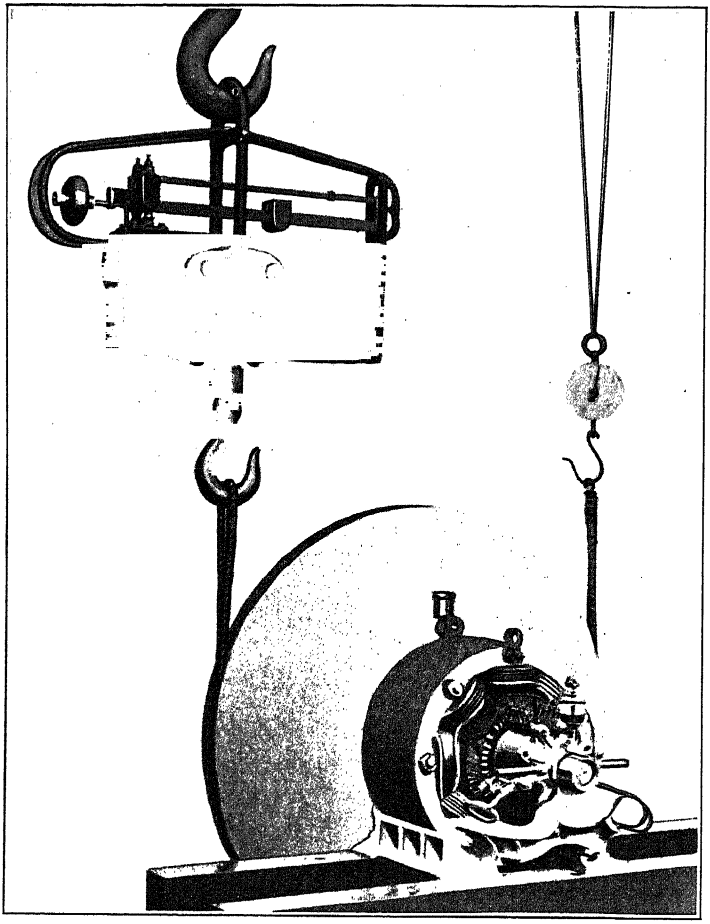


FIG. 33.

brake of W. Froude was also employed and by its means good results were constantly obtained.

ROPE BRAKES AT THE CENTRAL TECHNICAL COLLEGE, LONDON.

Fig. 34 shows the brake of the experimental engine at the Central Technical College. I am indebted to Prof. W. C.

Unwin, F.R.S., for the original drawing from which the figure was reduced. The brake wheel is of channel section (shown at

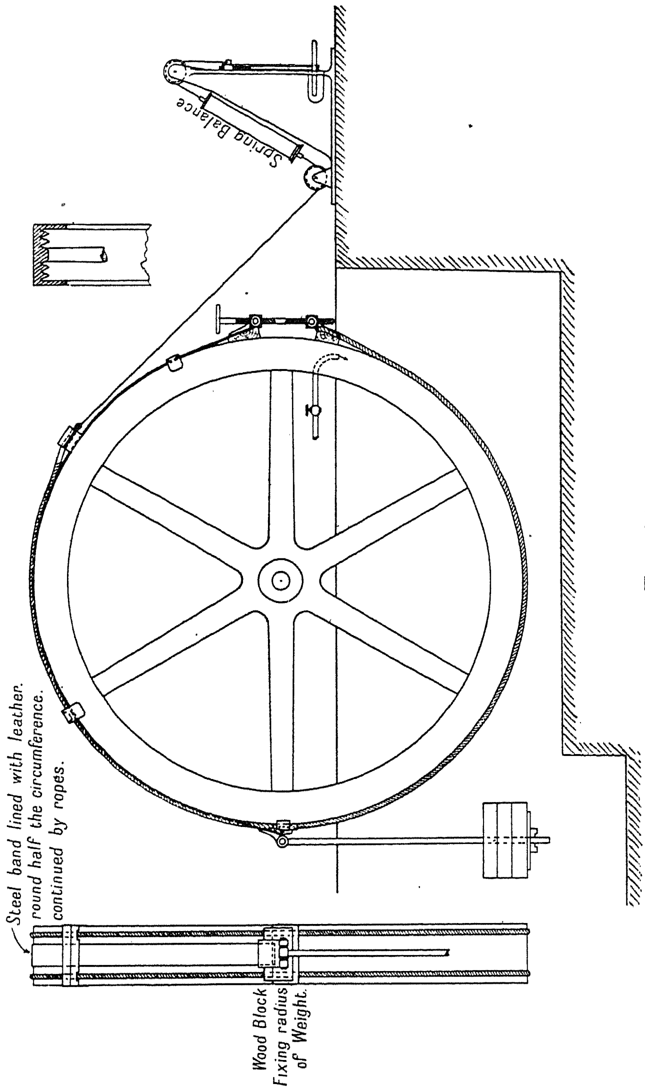


FIG. 34.

the top right side), and when in operation is constantly cooled by water fed into the channel. The radius at which the load acts is fixed by a wooden block shown on the left of the figure.

The spring balance is so placed that it can be readily adjusted by means of a screw and hand wheel. In another engine in the same institution, in which either of two wheels can be used as brake wheels, their respective diameters being 4 feet and 6 feet, the dead weight of 140 lbs. and spring balance are used with a single Manilla hemp rope $1\frac{1}{4}$ inch in diameter. The rim of the flywheel is cooled, and the power absorbed is up to 7 or 8 horse-power. In another rope brake made to absorb 12 to 15 horse-power the load is 200 lbs., and two ropes are employed, their ends being fixed to blocks from one of which the dead weight hangs; the other is connected to a spring balance by means of a single rope. The two ropes are kept in place on the face of the flywheel by three equi-distant blocks. In each case the effective radius at which the load acts is the distance between the perpendicular let fall through the centre of the flywheel and the parallel straight line through the centre of gravity of the load. No one watching these rope dynamometers when running can but be impressed by the simplicity of the device and its surprising steadiness for hours together. As long as the rope is dry the action is steady; but not so when lubrication is introduced—the spring balance then oscillates to such a degree that readings cannot be readily made.

A friction brake, formerly used at Cooper's Hill College, is illustrated in Fig. 35, reproduced by the permission of the editor of *Engineering*. Several useful additions to the usual brake are shown, as devised by Mr. James Hopps of the Mechanical Laboratory, Cooper's Hill College. The description is slightly abridged from *Engineering*, April 17, 1903.

The cooling water enters through the regulating cock A, passes through the flexible pipe B, and discharges into the channel of the rim of the wheel through C. The heated water is withdrawn through D. The pipes C, D, and D_2 are adjusted by means of the knurled knobs I and I connected to the worms and worm-wheels. The flow of both feed and discharge water can be accurately adjusted. By continuously depressing the collecting pipe D, the water can be quickly removed without flooding the engine-room floor. The best position and form of the supply pipe depends upon the velocity of the rotating wheel and the contained water; in the case before us the velocity of the innermost layer of water was 28 feet per second; by

curving the pipe C to the same curvature as that of the rotating water and placing the jet at about 30 degrees from the bottom of the wheel the waters blend without splashing.

The friction bands are applied on the top half of the wheel, and are loaded with a series of weights G, etc. The 50-lb.

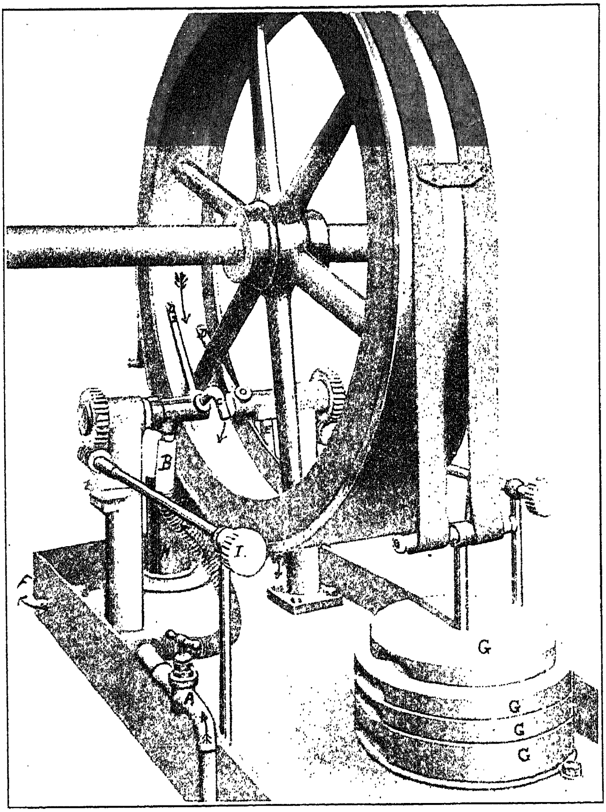


FIG. 35.

capacity counter-spring at the other end of the band is not attached to the floor, but to a weight of 30 lbs., which rests at the bottom of the tube H.

Should the engine suddenly slacken speed the weights G will descend about $2\frac{1}{2}$ inches, when they will come to rest, and the counter-spring will raise the 30-lb. weight through a corre-

sponding distance in the tube H. This prevents any overstraining of the counter-spring. Any danger that might arise from the weights G overrunning is prevented by a tail-pin with collar passing through the bottom of the tray.

The brake dynamometers of Thomson, Imray, Carpentier, Reckenzaun, and Raffard form a small class of machines, in which by different constructions the same principle of automatic government of friction is employed. In order to keep $e^{\mu\theta}$ (which equals $\frac{W}{w}$) constant θ is made to vary inversely as μ.

The Dynamometer of James Thomson.

On the shaft driven by an engine, or on the shaft of the engine itself, two pulleys of equal diameter are carried, one keyed to the shaft and the other loose on it (Fig. 36). The fast pulley is the brake wheel, while the function of the loose pulley is to carry a portion of the brake band or strap, which partly embraces both of the pulleys. The weights W and w are adjusted so as to balance the torque (a word invented by the late Professor James Thomson). Should, however, μ increase, then the loose pulley is rotated a little, and its movement lessens the angle θ embraced by the band on A and thus automatically keeps $e^{\mu\theta}$ constant.

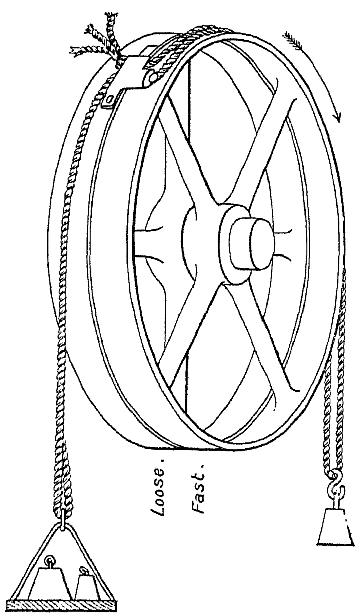


Fig. 36.

The Dynamometer of J. Imray (Figs. 37 and 38).

The automatic adjustment in this dynamometer is obtained by altering the arc of contact of the rubbing surface. The brake pulley A is partly embraced by a succession of wood blocks attached to a

flexible brake strap EGF. This is fixed to a counterpoised sector-shaped frame ECDD moving about a line in the axis of the shaft B, the radius of its arc being equal to that of the brake strap, one end of which hangs vertically and is loaded with a weight W; the other end also hangs vertically and is loaded with a less weight P. If the weights be adjusted so that the brake absorbs the power required and μ changes (decreases, for example), then the load falls a little, and in doing

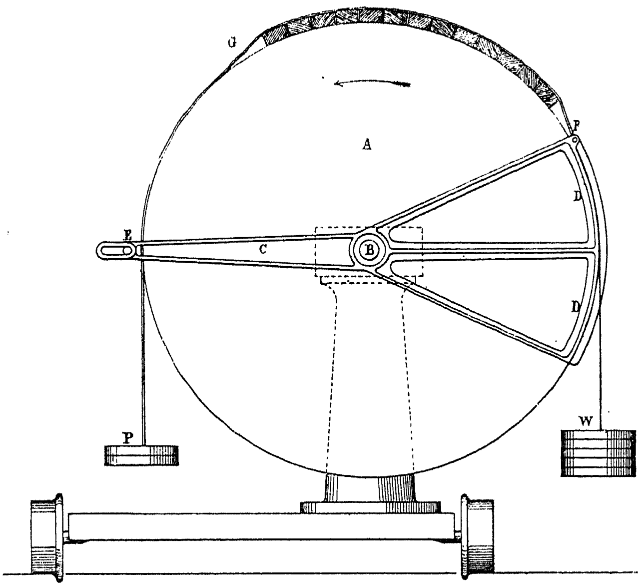


FIG. 37.

so causes the brake strap to bring a larger surface of brake block in contact with the pulley, thus restoring equilibrium until the next disturbance takes place. The genesis of the dynamometer of Imray is so good an example of mechanical evolution that I have quoted in full a passage which occurs in the discussion which followed a paper by Mr. W. W. Beaumont, Institution of Civil Engineers, Vol. XCV., 1888—89, Part I., p. 57, due to Mr. Imray, who said :—

"It was many years since the late Mr. William Froude and he investigated, at considerable length, the conditions of the frictional

hold of belts upon pulleys, and the result was communicated in a paper by Mr. Froude to the Institution of Mechanical Engineers (Proc., 1858, p. 92). The first thing they had to look at was this. At that time amongst engineers there was a fallacy prevalent that the larger the pulley the greater was the frictional hold of the strap upon it. They disposed of that by trying pulleys of all sizes from 5 inches to 5 feet, with straps on them loaded with weights, and there was not a shadow of difference between them. The diameter of the pulley had nothing to do with the frictional hold. They then investigated the question, and they thought that they were the first who had come to the formula for frictional hold, which was very much like the one given by Prof. Ayrton.

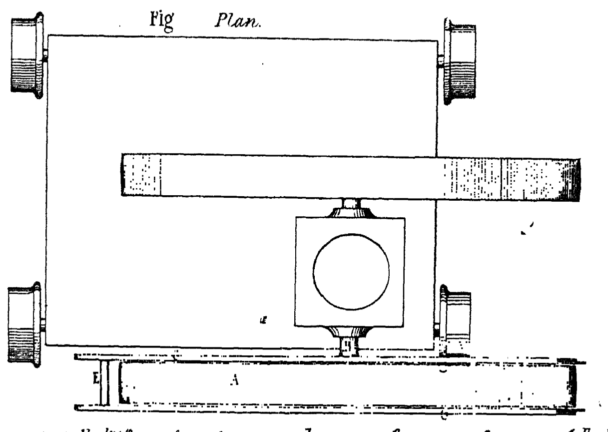


FIG. 38.

It appeared from that formula that a change in the number of degrees of the arc of contact made a great change in the frictional hold. For instance, they found that if one weight was 1 lb. and the other weight was 3 lbs., when half the circumference was embraced, then the latter would be 9 lbs. when the whole circumference was embraced, when three halves 27 lbs., and so on according to the formula. It therefore appeared to him that the best way of making a brake automatically adjustable was to make it alter for itself the amount of circumference embraced by the strap. For that reason he schemed the brake shown in Figs. 37 and 38. There were two arms, one on each side of the wheel. Those arms carried metal straps, by which the large weight was hung; and to the top of those arms at F the brake strap was attached. Whenever the weight rose it took a less part of the circumference; when it descended

it took a greater part of the circumference, so that it always cured itself, and it kept very steady. He believed that Mr. Froude used it, and to a large extent had found it successful."

THE DYNAMOMETER OF CARPENTIER (Bulletin Société des Anciens Elèves des Ecoles Nat. des Arts et Métiers, No. 186, 1880).

In this machine the automatic adjustment is obtained by utilising the same principle, but instead of a band a rope is employed, which is held at its mid-point by a hole in a flange projecting from the loose wheel. The rope is coiled twice round the pulley keyed to the shaft, and twice round the loose pulley. This dynamometer proved itself useful in finding the power delivered by small dynamos. Carpentier's dynamometer was remodelled and improved by I. Raffard, so that larger powers could be dealt with, such as 6 h.p. (Bulletin of the society quoted above, No. 212). In this machine three equal pulleys are carried on a shaft close together. The middle one is fast on the shaft and the two external pulleys are loose on it. A balanced bar shaped like an E, but without the small middle projection of this letter, is free to rock about its ends, which are centred on the axis of the shaft, on either side of the three pulleys. Three steel bands are attached to that part of the bar which faces the pulleys. The central band passes over the fast pulley in a contrary-to-clock-hands sense, and is loaded with a weight P. The two other bands are carried under the two outer pulleys in a clock-hands sense, and are attached to the end of a scale beam, so that they are loaded by means of weights suspended from the other end of the beam. When the central pulley is driven by the motor under trial, contrary-to-clock-hands, the arc embraced by the outer bands varies inversely as the friction of the central band. Thus compensation for change of friction is established, as in the brakes of Imray and Carpentier. The pulleys and the bands were partly immersed in a water trough. This dynamometer was improved by A. Reckenzaun, so that larger powers could be dealt with.

[Another way of making the effective friction of the belt of a brake change according to its position is described in the

Electrical World of New York, March 9, 1907, p. 520. This is attributed to Scheibe. An ordinary leather belt hangs over the brake pulley, with a light weight on the side in contact with the downward moving side of the pulley and a heavier weight on the other side. The difference of the two weights multiplied by the effective diameter is the torque. One half of the belt at the end carrying the greater weight is studded with copper rivets with their flat heads on the side next the pulley. The friction of these is so much less than that of the plain leather that the belt automatically takes its place where the balance is exact. An example is given showing a ratio of change of 1 to 10. Only small weights are contemplated.]

The Friction Dynamometer of Prony.

To Prony must be attributed the earliest method of testing the output of a prime-mover by means of a brake acting on a driven pulley. The Prony brake in its simplest form consisted of a pair of brake blocks partly embracing a pulley. From one of the blocks an arm projected which was loaded with a weight. In Fig. 39 a diagrammatic sketch is shown of the brake. The required pressure on the pulley was obtained by regulating the nuts above the lever arm. Let us suppose that a weight of 100 lbs. was suspended from the arm, and that its line of action measured horizontally was 6 feet from the axis, also that the pulley made 300 revolutions per minute; if there were no sliding of the brake over the pulley the weight would be drawn up just as if the rope were coiled on a pulley of 6 foot radius, so that the work is equivalent to 100 lbs. raised at the rate of $2\pi \times 6 \times 300$ feet per minute, and the brake horse-power is equal to

$$\frac{2\pi \times 6 \times 300 \times 100}{33,000} = 34{\cdot}3.$$

I am indebted to the manager of the *Engineering Magazine*, November, 1904, for the figure shown. In this ideal case the rope is supposed to have no thickness. For practical purposes this brake is usually made with a brake block on the under side of the pulley, and a row of small blocks attached to a flexible band embracing the upper half-circumference of the pulley; also the weight is suspended from an arc-shaped limb

which forms a part of the lever arm. This prevents the value of the load changing when the arm is deflected on either side of the horizontal position. The whole is counterpoised with a weight which can be adjusted. The capacity of this form of dynamometer for measuring power has been assumed to lie between 5 and 200 horse-power. Thurston designed a water-cooled brake dynamometer of the Prony type to absorb

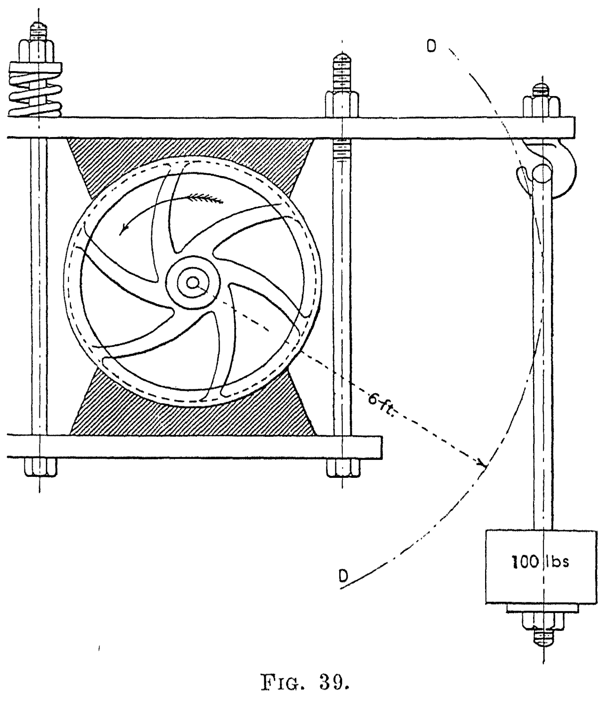


FIG. 39.

540 horse-power. The brake wheel was 5 feet in diameter, 2 feet over the face, and ran at 100 revolutions per minute. It was embraced by two brake straps lined with wood blocks 3 inches wide. The brake wheel was well lubricated with lard and plumbago. Something under 200 horse-power appears to have been absorbed by this dynamometer (Journal of the Franklin Institution, Vol. XCI., p. 290). The indications are good when the output of the engine or motor is fairly constant, but with varying powers the inertia of the arm

interferes with steady running and renders the indications difficult to estimate. An exhaustive note on the oscillations set up in this form of brake dynamometer will be found in the Proceedings of the Institution of Civil Engineers, Vol. XCV., pp. 41—47, by R. E. Froude. When instead of a brake block system a rope is employed to produce the required friction, a mass having a considerable moment of inertia is avoided as is its consequent oscillation and steady readings are easily obtained.

THE BRAKE DYNAMOMETER OF MR. COOPE (Proceedings of the Institution of Civil Engineers, Vol. XCV., p. 49).

In this machine the pulley is almost entirely embraced by wood brake blocks, attached to a flexible brake strap which was divided into two parts, each covering about one-third of the face of the pulley, and leaving an intermediate space between them which was filled by four cords, which partly embraced the pulley. Regarding the pulley as the face of a watch, the cords were attached to the brake straps at X, and were a tangent to the pulley at III. The greater of two weights was suspended from a cross bar connecting the two brake straps, and was raised when the pulley revolved in the clock-hands sense. The friction of the brake straps was regulated by a screw which connected its ends together. A smaller weight was suspended from the cords (four side by side) hanging from the other side of the pulley. If by any increase of friction the greater of the two weights was raised, the arc embraced by the cords was reduced, and consequently the friction. If the friction decreased, then more arc was covered by the cords, so that good compensation was the result. It was suggested that complete balancing would have been effected by hanging a group of the same sort of cords from the bottom of the two weights after the manner of a festoon.

I may notice that this would not be the case unless the hanging ropes were of infinite length. A rope of finite length hanging from two weights would approximately form a catenary curve, and the two weights would be drawn together and consequently their pulls would not be in vertical lines. If the hanging ropes passed round a pulley (in a block) of exactly the same diameter as the brake pulley, then balancing would

exist. In this machine the method of compensation is somewhat similar to that employed by James Thomson. The details, however, differ materially from those of Thomson's machine.

The Friction Brake of Appold and Amos.

This brake was primarily designed for controlling the rate of pay-out of the French Atlantic cable before the year 1858. The pulley to be controlled was embraced by a brake band, and its ends were attached to two points in the lever shown in Fig. 40. At the end of the lever was a short slot A engaging with a fixed stud; if the friction between the band and the pulley increased, the lever was moved to the left, and the brake band slightly released thereby and the friction reduced. The brake maintained a uniform tension on the cable while it was being paid out. This brake when employed as a dynamometer by the Royal Agricultural Society for testing engines was somewhat modified. The driven pulley was embraced by a flexible band lined with numerous wood brake blocks.

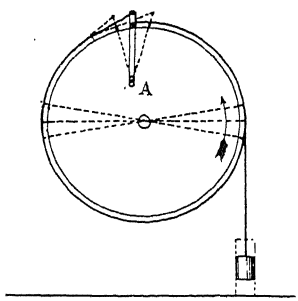


Fig. 40.

The toggle lever was placed at the lowest part of the circumference of the pulley, a weight was suspended from a hook projecting from the brake band in line with a horizontal diameter, and by means of a regulating screw the required friction was set up. The correct working position of brake band was indicated by a pointer and the readings taken when in this position. The brake used for paying out purposes appears to be excellent, but considerable doubt has been felt as to its correct performance as a dynamometer. The toggle lever introduces conditions which if not entirely accounted for might cause error. The brake is certainly interesting from a historical point of view. Experience has shown that the utmost simplicity must be aimed at in connection with the automatic regulation of the friction brought into play.

THE BRAKE DYNAMOMETER OF BALK.

In this machine a compensating lever is employed, connected to a flexible band lined with wood brake blocks, as in the Appold brake, but in this dynamometer the lever is situated outside the circle of the pulley, and its external end is made with a slot which is large enough to allow of some play between it and a fixed pin. To the end of the lever, in line with the pin, a scale pan is suspended. From the face of the brake strap most distant from the lever a weight hangs; rotation contrary-to-clock-hands tends to lift this weight. The initial tension of the brake strap is regulated by means of a left and right hand screw which connects its ends. When the machine is working the load in the scale-pan is adjusted so as to keep the lever floating, and not touching the pin which passes through the slot. The effective moment equals $Pr_1 - pr_2$ and the horse-power absorbed equals $\frac{2\pi N(Pr_1 - pr_2)}{33,000}$. The blocks which lined the brake strap were made of beech or plane-tree wood. The pulley and blocks were well greased, and it was found that with but little attention a run might be made for a whole day. The dimensions of one of these brakes, used by Messrs. Ransomes, Sims, and Jefferies, were as follows:—Diameter of brake wheel, 6 feet; width, 1 foot, load suspended from flat steel spring tapes at a radial distance of 3·183 feet. By means of a lever at the side of the pulley a counter could be thrown into gear.

A WATER-COOLED BRAKE DYNAMOMETER (by Messrs. R. Garret and Sons).

In this machine a brake strap lined with beech-wood blocks embraced the driven wheel. The strap was compensated for friction by the Appold lever, and the loading and method of tightening are the same as in that brake; but an important improvement was introduced, namely, a water channel in the rim of the brake wheel. Water was introduced by means of a tube which dipped into the channel, which it left by evaporating. The rim of the wheel was thus kept cool and also the brake blocks.

[THE GRIFFIN ENGINEERING COMPANY'S BRAKE.]

[A neat and portable absorption dynamometer constructed by the Griffin Engineering Co., Ltd., of Bath, is described and illustrated in *Engineering*, February, 1912, p. 572, and also in *Internal Combustion Engineering*, January 7, 1914. Lignum vitæ shoes may be pressed against the interior conical faces of

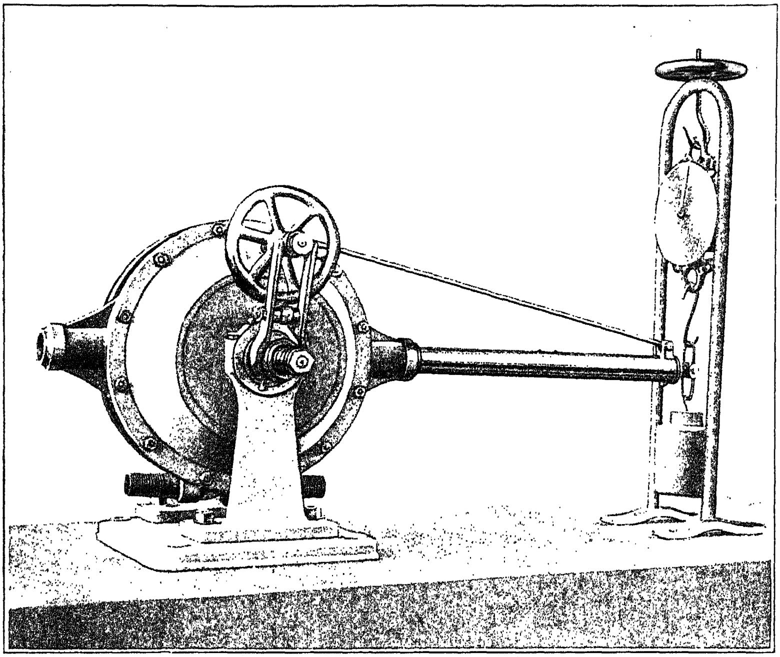


FIG. 41.

the friction drum by means of a stationary hand-wheel through the intervention of mechanism the nature of which is made sufficiently clear in the illustrations (Figs. 41 and 42), for which I am indebted to the makers. Water is passed through the interior to keep it cool. The direction of rotation is such that the long arm, which is stayed as shown, tends to lift the weight at its end. The force there exerted is measured by the aid of a spring balance, and as usual the horse-power is known when the speed of rotation, the length of the arm, and the force at

its end are known. A socket is provided on the opposite side of the casing to take the arm when it is desired to test the horse-power of a shaft running in the opposite direction. This appears to the writer to be a particularly neat and convenient form of dynamometer, and though included among the friction brakes it might equally well have been described in the next chapter among fluid friction brakes, for it is only when being used at its highest power or at low speeds that solid friction is made use of. For ordinary use at high speeds the shoes are removed out of contact with the friction surfaces by means of the hand-wheel, and "water attrition" alone is relied upon for taking up the load. It is on this account that this dynamometer is especially suitable for testing petrol motors, for the higher index law of fluid friction, as explained on page 226, is necessary to obtain stability of speed with this type of motor. The makers recommend that water should be supplied at the rate of about five gallons per brake horse-power hour.

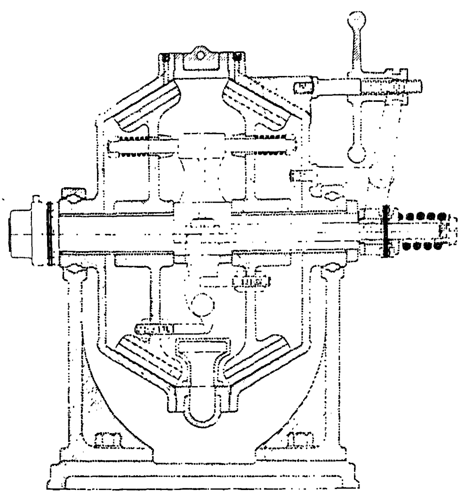


FIG. 42.

The inlet and outlet water pipes are shown at the bottom of the casing. The distance between the shoes and the casing or the pressure between them may be regulated by the hand-wheel and the resistance adjusted while the load is on. This dynamometer is convenient in that the water supply may be taken from a tank very little above the dynamometer level, as no hydrostatic pressure is required to overcome internal pressures. According to tests made in the presence of the representative of the journal, *Internal Combustion Engineering*, on a dynameter made for the Hong-Kong University, the indications of this dynamometer are unusually steady as compared with those of an ordinary friction brake. Four sizes are at present made.

No. 3, with discs 21 inches in diameter, is suitable for powers ranging from 5 horse-power at 250 revolutions per minute to 70 or 80 horse-power at 3,000 revolutions per minute.]

[ALDEN ABSORPTION DYNAMOMETER.]

[There is an account of a very large Alden absorption dynamometer in the *Electrical World* (New York) of October 31, 1908, p. 945; see also Trans. Amer. Soc. Engineers, Vol. XI. A central cast-iron plate keyed to the shaft runs in a casing which is free and the torque of which is measured. Copper plates attached to the casing lie one on either side of the cast-iron plate. Water is fed into the casing at any desired pressure and circulated, and according to the pressure so the friction between the copper and the iron may be varied. Oil is circulated between the friction surfaces. The power that can be absorbed is limited by the heat which can pass through the copper plates. "Since the maximum peripheral speed should not exceed 7,000 feet per minute and the best friction surface is between 6 and 10 square inches per horse-power, the large capacity brakes require more than one disc." The figure shows a 60 inch four-disc brake in the bearings of a pulp grinder and directly connected to the water wheel. It absorbs 3,000 horse power when operating at a speed of 225 revolutions per minute. The dynamometer is given an automatic control, depending on the regulation of the water pressure, by the slight movements of the casing.]

[NICHOLSON'S LATHE TOOL DYNAMOMETER.]

[In the Proceedings of the Institution of Mechanical Engineers, June, 1904, pp. 883—925, there is an interesting and important paper by Prof. J. T. Nicholson, of the Municipal School of Technology, Manchester, entitled "Experiments with a Lathe Tool Dynamometer." Reference is made to the few previous investigations on the same subject. Hastig published a work in Leipzig in 1873. Mr. A. Mallock published in the Proceedings of the Royal Society, December, 1881, a paper on experiments which he had made in the engineering workshop at Cambridge. Prof. R. H. Smith, in his work on Cutting Tools

published in 1882, gave the results of experiments which he had made.

In the second and more complete lathe tool dynamometer of Prof. Nicholson the tool is supported so that it can move about both a vertical and a horizontal axis at the back end, being supported immediately under its cutting end by means of a powerful lever or resting upon a knife edge in the middle, while the vertical force due to the cut is transmitted by the strut which supports the cutting end of the tool and which rests upon a knife edge at one end of the lever. The up thrust at the other end of the lever, which is at the back of the lathe, is similarly transmitted by a knife edge and strut to a diaphragm gauge filled with boiled distilled water. The water communicates by a pipe with a Bourdon pressure-gauge. By this means the vertical force on the tool can be ascertained, and at the same time the movement, even with forces as great as 15 tons, is excessively small. A corresponding device measured the traversing force needed to make the tool follow the feed, and a third measured the radial force necessary to keep the tool up to its work. Experiments were made with tools of various shapes taking off shavings of mild steel or cast iron of different thicknesses and breadths and at different speeds. The power required to cut mild steel or cast iron is the same and is just over 2 horse-power per pound per minute, while provision must be made for a vertical force of about 100 tons per square inch of section of shaving. In addition to this much information of importance as to the tools and their durability and the power required with different cutting angles and also as to the forces which must be met in the design of the lathe was obtained. Other experiments with the same object were referred to in the discussion. As there is no provision in the scheme of the book for this I have put this among the absorption dynamometers, for strictly speaking it is one.]

CHAPTER V

WATER BRAKES

Historical.—G. A. Hirn,* while working on the mechanical equivalent of heat, devised a method of finding the heat produced by the friction of water, in which he employed concentric tubes ; the inner one rotated, while the outer concentric one was free to rotate, but in doing so raised known weights. This appears to be the prototype of hydraulic dynamometers, in which some rotating internal organ imparts motion to an outer casing through a liquid connecting medium. The work of Hirn is of such importance and interest that I give here a translation of his description of the apparatus.

" *Friction of Water.*—In studying this I have employed apparatus constructed thus. Firstly, a polished brass cylinder 30 centimetres diameter and 100 centimetres long, mounted on an axis connected to a motor having a very regular rate of rotation, and capable of a variation of speed between 60 and 600 revolutions per minute. Secondly, a fixed (external) cylinder, polished inside, placed concentric with the former internal one and distant from it by 3 centimetres. Discs furnished with stuffing-boxes, through which the axis of the internal cylinder passed, formed the ends of this external cylinder. The whole space between the two cylinders could be filled with any liquid, which was prevented from leaking out by the stuffing-boxes.

" When the internal cylinder rotated in either direction, the friction

* "Théorie Mécanique de la Chaleur," 2nd ed., 1865, p. 65, and 3rd ed., 1875, p. 92.

which its external surface exerted upon the water, and which the water in turn exerted on the internal surface of the external cylinder, tended to rotate it about itself. Two parallel levers fixed to its two ends carrying balance pans allowed the rotation to be checked by loads which showed the value of the friction. The 'tare' of the levers and the value of the friction due to the stuffing-boxes were easily found by making the internal cylinder rotate very slowly in opposite directions. By means of two vertical pipes fixed as close as possible to the stuffing-boxes a continuous current of water under perfect control could be passed through the apparatus. The temperature of the liquid was taken as it entered and also as it left the apparatus. As far as possible the temperature of the water entering the apparatus was kept as many degrees below the temperature of the room as the water leaving the apparatus was above it. The law of cooling of the apparatus was carefully determined, so that it was easy to make the necessary corrections, which were always small. This apparatus, as a whole, constituted a veritable liquid friction balance. It enabled one to learn the work expended on this or that liquid for any speed, and also the calories produced by this friction, in a liquid whose specific heat was known.

"Owing to the large size of the apparatus and the speed which the internal drum received, this apparatus allowed of a considerable amount of mechanical work being employed—750 kilogram-metres per second, or 10 horse-power (French measure).* I insist on the importance of this feature, as a guarantee of the exactitude of the numbers found. The results deduced have been satisfactorily regular. Six experiments using water and various speeds, and different amounts of liquid introduced per second between the two drums, have given me 432 kilogram-metres as the *work* which produced one calorie, and hence the value of the heat equivalent. The water friction experiment took its origin from Joule, and after him Favre. The values obtained by these two experimentalists differ a little from those I have given (Joule found it to be 424 kilogram-metres)."

Recent determinations make the value 428.

There is an interesting experimental method of finding the friction between a liquid and a solid, which may be considered in this connection. It is due to Prof. J. Perry, F.R.S. I give here a sketch of the method, which is described more fully at pp. 76—78 of his "Applied Mechanics," 1897. It has to do with the resistance to motion of water in a pipe or the resist-

* 75 kilogram-metres per second = force de cheval. The horse-power English = 1·01386 force de cheval.

ance to the steady motion of a ship. Usually the motion between a liquid and a solid is complicated. The simplest motion is in parallel layers. We may imagine two infinite parallel boundaries with the fluid between them, one at rest and the other moving with a uniform V, and that the fluid adheres to each boundary. Let the distance between the two adjacent surfaces be b, then the tangential force per unit area required to keep up motion is $\frac{\mu V}{b}$. μ is the coefficient of viscosity. Theoretically μ should be constant, if the motion is in truly plane layers. Since an experiment with infinite surfaces is impossible, the condition required was approached by employing the apparatus shown in the figure (Fig. 43). FF is a hollow cylinder so supported that it cannot move sideways. Resistance to rotation is opposed by a torsion wire A, by means of which it is suspended. This cylinder dips into an annular space between two surfaces filled with liquid and wetting all the surfaces. When the vessel EDDE is rotated about its axis the liquid moving past the cylinder F tends to rotate it. The torsion wire A resists this torque due to friction, and the value of the twist of the wire shown on a scale becomes a measure of the viscosity of any liquids which may be experimented on. The apparatus of Professor Perry was designed and partly constructed in Japan in 1876. An important paper by him will be found in the Proceedings of the Physical Society, London, Vol. XII., pp. 236—255. The following facts were ascertained. At constant temperature, below a certain critical speed, experiment showed that friction was proportional to velocity; so that μ could be found. The law changed at the critical speed, and above it friction was seen to be proportional to a higher power of the speed than unity. μ was found to decrease rapidly with increase of temperature. For infor-

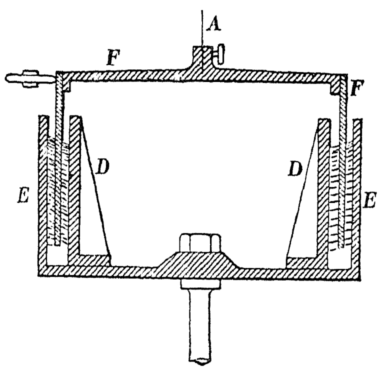


FIG. 43.

mation on critical speed, see Philosophical Transactions of the Royal Society, Part III., 1883, Osborne Reynolds, F.R.S. It would be foreign to my subject to give exhaustive references to researches on friction due to fluids in motion. I have introduced this interesting experiment, since by its means the properties of the kind of friction generated in the apparatus of the type of that employed by Hirn were determined.

In 1876 O. Reynolds, while experimenting on a multiple steam turbine at speeds of 12,000 revolutions per minute, employed a water brake, or, in his words, "having a centrifugal pump suspended on the shaft and working into itself," the head against which the centrifugal pump was working being regulated by a valve situated in the external circuit of the water. An account of this apparatus was given before the Mechanical Section of the British Association in 1887. It was at this meeting that the paper on the water brake of William Froude was also given. In both machines the resistance to turning was regulated by adjustable sluices arranged to cut off the passage of the liquid within the casing. Reynolds remarks that "Mr. Froude invented an internal arrangement which affords a resistance out of all comparison with any other form."

For the exact details of O. Reynolds' hydraulic dynamometer "Scientific Papers of O. Reynolds, F.R.S.," Vol. II., pp. 353—359, should be consulted. I give a rather detailed account of the remarkable water brake of William Froude, and a description of the modern form of the brake designed and made by Messrs. Heenan and Froude, which has been successfully employed in testing engines of considerable horse-power, in some cases reaching 2,000 B.H.P.

The Turbine Dynamometer of Froude.

In July, 1877, a very remarkable paper was read before the Institution of Mechanical Engineers at their meeting at Bristol, the title of the paper being "On a new Dynamometer for Measuring the Power delivered to the Screws of Large Ships," by Mr. William Froude, F.R.S. The original paper should be carefully read to appreciate the genius of William Froude. In hydrodynamics it stands out as a monument to mathematical

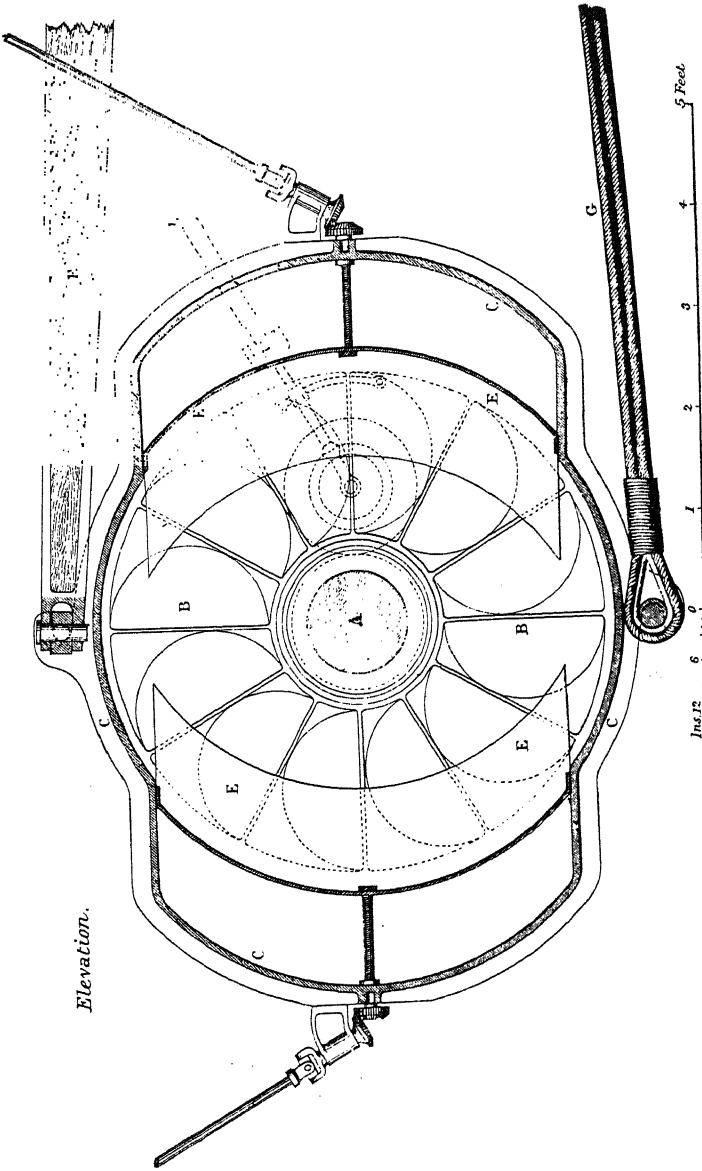
acumen and its practical application. The following abstract will perhaps be of use to those who are unable to obtain the original paper.*

For the measurement of fairly small powers the friction-brake dynamometer is effective and simple, but serious difficulties arise when the horse-power to be absorbed is great and of the order of thousands instead of tens of horse-power. In the case of a friction-brake work-measuring machine the engine in delivering its power will be virtually winding up a weight out of a well of indefinite depth ; but the weight, instead of being constant, will vary with the speed of rotation, just as the resistance of a propeller does ; and so the work done by the engine tested will more closely resemble its natural work, and the same circumstance renders necessary some method of recording the changes of resistance occurring during the trial. Instead of the continuous friction due to two surfaces in contact, it will be seen that the total reaction will be due to the impact of fluid streams maintained in a state of intensified speed by means of a sort of turbine rotating within a casing full of water ; the turbine and the casing are mounted on the screw shaft in place of the screw, and while the turbine rotates the casing is held stationary by means of a lever pressing against a spring.

"The jets are alternately dashed forward from projections in the turbine against counter-projections in the interior of the casing, tending to impress forward rotation upon the casing, and are in turn dashed back from the projections in the casing against those of the turbine, tending to resist the turbine's rotation. The important point is, that the speed of the jets is intensified by the reactions to which they are thus alternately subjected ; and thus in virtue of this circumstance a total reaction of very great magnitude is maintained within a casing of comparatively very limited dimensions."

The construction of the apparatus is shown in Figs. 44—51. In Fig. 47, A is the screw end of the screw-shaft, BB the section of "the turbine," which consists of a disc with a central boss, keyed to the screw-shaft. The disc is shaped into a channel of semioval section extending round the whole circumference. A

* I am indebted to the Institution of Mechanical Engineers for their kind permission to reproduce parts and figures from the paper mentioned.

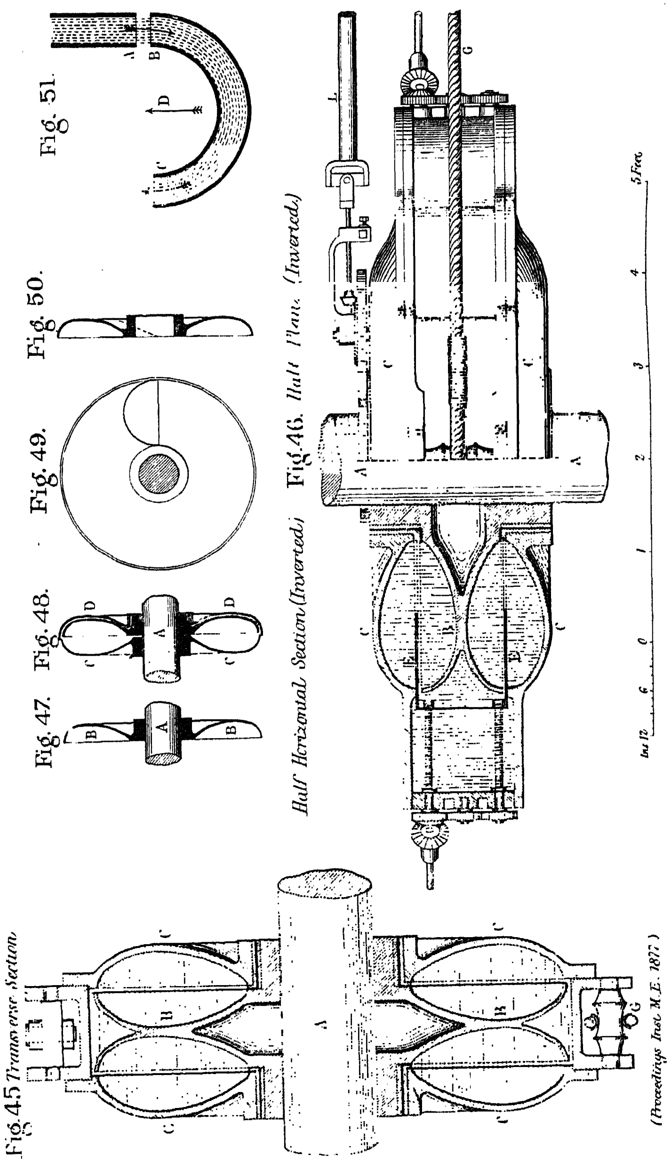


Elevation.

FIG. 44.

(Proceedings Inst. M.E. 1877.)

turbine disc for dealing with 2,000 indicated horse-power is 5 feet in diameter. Fig. 48 shows " the casing " CC, DD, the



former representing the front, the latter the back; its face is also shaped into a channel, a counterpart of that of the turbine

disc. The semioval channels nearly touch, and in effect form one complete oval channel, though the halves are separated by an imaginary plane of division. The boss of the casing is an easy fit over that of the turbine ; thus the turbine carried by the shaft can revolve within the casing without touching it, while the casing itself is stationary ; and one half of the oval channel is rotating while the other half is at rest. The two half-channels are not unobstructed, but they are each cut across by a series of diaphragms, as shown in Fig. 49, in which a single diaphragm is drawn. The diaphragms are semicircular in outline, so that when set obliquely their circular edges are in contact with the bottom of the channel and their diameters span the major axis of the oval.

One of the diaphragms is shown in Fig. 50 end on. Each half-channel has twelve of these diaphragms, dividing it into a series of cells, two half-cells together making one complete cell with circular outline. The oval channel may be regarded as a series of obliquely placed circular cells.

" As the function of the turbine is to rotate while the casing remains at rest, one half of each cell is moving past the other half in such a manner that the moving half, if viewed from its stationary counterpart, would by reason of the oblique direction of the diaphragms which form the cell sides appear to be advancing antagonistically towards it ; indeed, the motion virtually constitutes such an advance, because the bottom of each moving half-cell is continuously growing nearer to the bottom of the stationary half-cell which it faces.

" The effectiveness of this combination to resist rotation will be seen to depend essentially on the *quasi*-antagonistic virtual approach of the moving to the stationary half-cells. The channel and casing is filled with water. When the turbine rotates, the water in each of its half-cells is urged outwards by centrifugal force ; and subject to this impulse it forces inwards the water in the half-cells of the stationary casing, and so a continuous current is established—outwards in the half-cells of the turbine, inwards in the cells of the casing.

" The current originated by centrifugal force only, when once started possesses a power of growth independent of centrifugal force, but dependent on what has been called the virtually antagonistic motion of the two sets of diaphragms, and the cells of which they are the boundaries. The nature of this power of current-growth is discussed in an appendix. It was found that, with any given speed of the turbine, the system of internal motions gives rise to a speed-producing power called ' potential,'

which will continuously increase the speed of the currents up to the point at which the friction experienced by them when traversing the cells produces a resistance which equals the potential. The frictional resistance and also the potential are both proportional to the speed of the turbine, so that the speed of current is directly proportional to the speed of the turbine simply. It is not difficult to trace the manner in which the established currents produce the dynamometric reaction. The result is not affected by the slight departure from truly cylindrical form of the cells. Each of the circular discs of water will constitute a sort of vortex. The mode in which reaction takes place is clearly described thus."

Now each vortex in virtue of the centrifugal force, which is continually tending to stretch it edgeways, pushes against its circumferential boundaries ; and as these boundaries are in fact made up of the bottoms or circular outlines of the two half-cells occupied by the vortex (the one in the stationary casing and the other in the rotating turbine), the resultant force, measured in the plane of rotation of the turbine, is constantly tending with a determinate force to stop the rotation of the turbine and create rotation in the casing.

The magnitude of this force may be expressed as due to the reversal of the sum of the momenta of the vortex streams, measured in the plane of rotation of the turbine ; since streams when entering a cell are flowing in one direction, and in the opposite direction with the same velocity when leaving it, and the force due to this reversal is directly proportional to the momentum reversed per second, this equals the product of the mass acted on per second and the change of speed imparted to it in the plane of rotation of the turbine ; also the mass acted on per second varies as the mean speed of the vortex current, and this depends on the speed of the turbine : thus the tendency of the vortex to resist the rotation of the turbine and to rotate the casing is as the square of the speed of the turbine. Even if the turbine were suddenly stopped, the "vortical rotation" would continue until extinguished by friction.

There is yet another element of reaction existing only when the turbine is rotating. It is due to the fact that the hoop-shaped streams of the vortex stream is constantly sheared by the passage of the planes of the diaphragms of the turbine and also of the casing. The effective stream-speed is not changed

by this, since owing to the incompressibility of water each imaginary pipe must everywhere be traversed at the same speed ; but, from the action, the particles which form each stream at the points of shearing must be subjected to alternate changes of speed in the plane of rotation of the turbine. In passing from the stationary casing to the turbine cells, they assume the speed of the turbine in its plane of rotation and thus react on the diaphragms of the turbine with a definite force, proportional to the amount of momentum per second imparted to them as they pass. Again, in passing from the turbine cells to the cells of the casing, they lose that speed in the plane of rotation of the turbine, and so act on the cells of the casing, tending to push them forward with a force equal to that reaction which tended to stop the rotation of the turbine cells, the same mass being acted on each second in each instance, and as the same speed is in one case added and in the other deducted, the force is the same. Again we see that the reaction varies as the square of the speed of rotation of the turbine, since the momentum generated per second, causing the reaction, varies as the product of the mass operated on per second and the speed imparted to it ; the speed is that of the turbine, and the mass operated on varies as the speed of "vortical rotation," which is as the speed of the turbine.

The conclusion arrived at from the theory of the turbine dynamometer by Froude was that "their respective moments of reaction, with the same speed of rotation in each, should be as the fifth powers of their respective dimensions." Experiment showed that this deduction was true. Two similar water dynamometers were made, in which the diameters of the turbines were 12 and 9·1 inches and $\left(\frac{12}{9{\cdot}1}\right)^5 = 4$, so that at a given speed of rotation of the turbines the ratio of the moments of the two machines should be 4. The ratio turned out to be 3·86. The small difference is attributed to the fact that in the larger of the machines the internal friction was rather less in proportion than that in the smaller one.

The analysis and the final deduction enable the engineer to design a turbine dynamometer to deal with horse-power of large value. The dynamical principles of the machine are clearly

set forth on pp. 252—260 of the Proceedings of the Institution of Mechanical Engineers, July, 1877. I have introduced this perhaps rather intricate description of the Froude turbine dynamometer, as at the present time (1910) a dynamometer designed on the Froude principle by Messrs. Heenan and Froude has been employed to "brake," a steam engine of 1,500 brake horse-power, built by Messrs. Browett Lindley & Co. The dynamometer was capable of absorbing 2,000 brake horse-power at 250 revolutions per minute.

The paper by Froude terminates with an illustration of the way in which the acceleration of the water stream is produced, and also by an illustration of the same principle by the late Sir Frederick Bramwell. The statement that "for two strictly similar but differently dimensioned instruments, the respective 'moments of reaction' with the same speed of rotation in each would be as the fifth power of their respective dimensions" is arrived at thus :—

Let two dynamometers, A and B, be compared, the diameter of A being double that of B, the revolutions per minute of both the same, the linear velocity in A would be double that of B, and from this cause the resistance would be as the square. In A the area acted on would be four times as great as that of B, or as the square of the increase in dimensions ; thus we should have four times the resistance acting on four times the area in A, and therefore the effective resistance is proportional to the fourth power of the increase of dimensions. Also the resistance would be opposed at the end of an arm of double the radius ; so that finally the power-absorbing capacity of the dynamometer would be proportional to the fifth power of its linear dimensions.

The operation of making a test with the machine is carried out thus. The boss of the turbine is bored out to a diameter larger than any shaft to which it will have to be applied and fixed to the shaft of the engine to be tested by means of an "adapter" which fits both the shaft and the boss of the turbine. The turbine so mounted will run true on the engine or propeller shaft. In the case of testing the engines of a ship the machine is attached to the shaft when the ship is in dry dock, and the casing connected to a supply of water which flows through the machine slowly and keeps down its tempera-

ture. By attaching a lever to the casing so that the ratio of the pressure at its outer end is as 1 to 10 a convenient pressure at the end of the lever can be dealt with; the force at the end of the lever acts on a horizontal flat steel spring supported at its ends, such that its maximum deflection is about $1\frac{1}{2}$ inches.

Experiments extending over many years have shown that for large loads the flat steel spring is greatly superior in constancy of action to the spiral spring, but recently spiral springs of excellent quality have been produced by a new process for purposes of weighing. But to return to Froude's paper: the movement of the end of the lever is communicated to a bell-crank lever, to the vertical arm of which a long connecting-rod is attached; to this is fixed a recording pen, which moves freely along a sheet of continuous paper which derives its onward motion from the engine shaft. Also a stationary pen traces a zero line, such as the lever pen would trace when at no load. The area of the diagram is the product of the moment on the casing and the speed of the shaft, or the *work* delivered by the shaft. The lever is also connected, if desired, to an integrating apparatus of the Ashton and Storey type. My own experience has led me to use, whenever possible, the diagram method of recording *work*, since the diagram shows not only the total work done between fixed limits, but it shows *how* the *work* is developed at each instant.

Either record by the addition of a time trace (usually made by a pen controlled by a clock) can be converted from a record of *work* to that of *horse-power*. In the machine described the excellent method of carrying the recording apparatus on points immediately over the supports of the ends of the spring ensure it against any external movements. A perfect trace is thus obtained, the ordinates of which are strictly proportional to the deflection of the spring, apart from any deflection of the frame by which it is carried. While the records are being made by the apparatus series of indicator diagrams taken from the engine give the indicated horse-power, and these when compared with the diagrams of brake horse-power show the amount of power expended between the cylinders and the end of the shaft. This difference or waste of power would exist

when the engines were propelling the ship, with the exception of friction due to thrust and a difference of friction on the bearings due to the propeller compared with that produced by the weight of the water dynamometer. Both of these items can be corrected by calculation.

The extinction of, say, 2,000 horse-power is accompanied by the heating of the water in the casing, and it has been shown by the author of the paper that the temperature would be kept below the boiling point of water if in each minute 8 cubic feet of cold water replaced the same amount of hot water in the casing; this passage of the water would in no way interfere with the dynamometric action of the apparatus.

It is almost necessary, even at the present date, to state in detail the great advantages which may be derived from the exact dynamometric tests of all classes of engines and prime-movers. There is a certain rather large and unknown quantity of power expended in friction which with the results of compression and release are very difficult to estimate by any method other than the dynamometric one. William Froude incorporates in the quotation I give here the real and important function of the test to be considered in this book. When discussing the difficulties which arise in three cases connected with ship propulsion, he writes :—

> "(1) The speed attained by a given ship, driven by a given indicated horse-power, fails to measure discriminatively the merits of the ship.
>
> "(2) No means exist of ascertaining which type of engine delivers the largest proportion of the power that it indicates.
>
> "(3) No test exists by which it is possible to measure concisely the specific constructional merit of this or that engine, or to determine the relative constructional merit of the engines supplied by different firms.

"The dynamometric test would remove at once each of these difficulties by substituting a final and real test for a collateral, and to a large extent delusive, one. For to rely exclusively on the test furnished by the indicator is almost equivalent to testing the power of a horse solely by the quantity of food he consumes and digests, or the efficiency of a boiler solely by the quantity of coal per hour it will legitimately consume on its fire-bars."

The following explanation (which has been slightly abbreviated) was given of the way in which current-growth was generated in the cells of the machine. Imagine (Fig. 51) a jet of water issuing at A with a velocity of 10 feet per second, to be caught by a fixed curved tube BC of the same diameter as A bent to a semicircle; the water would enter B at 10 feet per second and also leave B at the same velocity, and in passing round the bend from its centrifugal force would set up a pressure tending to move the bent tube away from the jet. Next imagine the bent tube to move as shown by the arrow D at the rate of 1 foot per second; then the water would be entering B at 11 feet per second relatively to the bend, and would leave the bend at this velocity relatively to the bend, but at 12 feet per second with respect to any fixed point; so that the forward motion of the bend at 1 foot per second would accelerate the flow at C by 2 feet per second. Now if the water leaving C entered a fixed semicircular bend, it would travel through this bend at 12 feet per second, and it might be again accelerated by passing through a second moving bend; and so on *ad infinitum*, that is, if it met with no resistances due to friction of any kind. In this illustration we may regard the moving bend to represent a half-cell of the turbine approaching a half-cell in the casing, so that as the water was discharged from the cells of the casing into those of the turbine it was subject to a constantly-increasing acceleration, which is only balanced by resistance due to friction in the cells equal to the speed-producing power. The force resisting the rotation of the turbine equalled the resultant, in the plane of rotation, of the centrifugal force due to the current as it traversed the curved contour of the turbine's cells; this equalled the force exerted on the cells of the casing in an opposite direction.

The Froude Water Brake Dynamometer (by Messrs. Heenan and Froude, Manchester: Figs. 52 and 53).

The principle involved in the action of this dynamometer has already been described. It will be remembered that vortex motion is established in each cell of a turbine of peculiar construction and maintained by its rotation. Since the heat produced by the work done on the water is considerable, cold

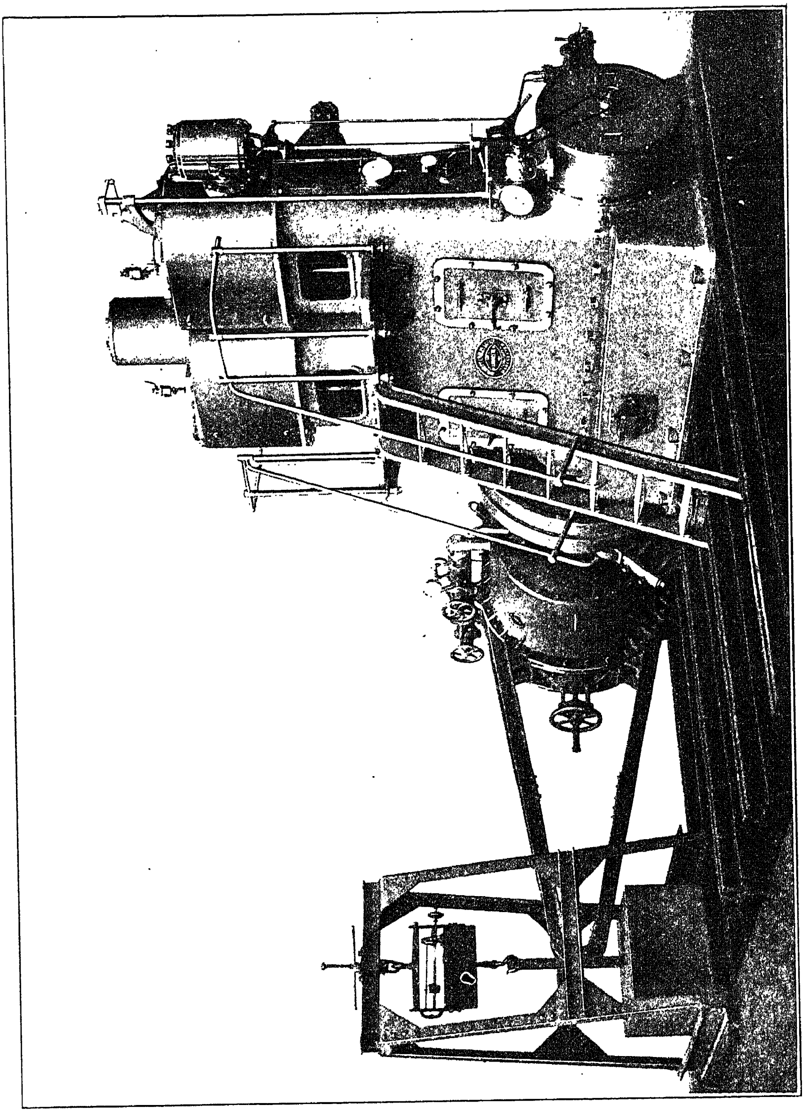


Fig. 52.

water has to be introduced, but in such a manner as not to disturb the vortex motion. To effect this water is admitted through the fixed vanes of the turbine to points situated in the centre of the vortices, where both the pressure and the velocity

are low. The lodgment of air is prevented by the water being introduced at a pressure of between 15 lbs. and 20 lbs. per square inch. The supply of cold water comes from a channel behind each set of the fixed vanes, and the hot water escapes into the outer part of the casing, leaving it by an outlet pipe placed at the highest point to enable air or steam to get away easily. The balance of the casing is not affected by the external connecting tubes, since they are flexible. The ability of the dynamometer to absorb power depends on the surfaces of the

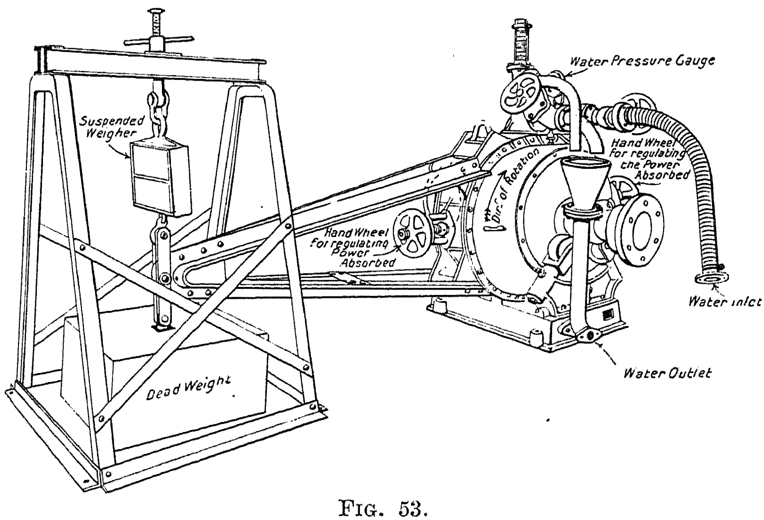


FIG. 53.

cells being as smooth as possible, so that the velocities of the water within the cells may be the greatest. In order that the power-absorbing quality of the machine may be under control, a thin metal shield can be interposed between the faces of the turbine and the casing so as to reduce the vortical action. By this means the power may be reduced to about one-fourteenth part of the maximum power; and in modern machines the power may be reduced to about one-fortieth of the maximum at any particular speed. The weight of the casing is not carried by the shaft, but by antifriction rollers, which can be adjusted so as to bring the turbine shaft into exact adjustment with the shaft of the engine to be tested. The casing is furnished with a lever weighted at its end with a weight greater than that which

is actually required, the portion of the weight not lifted by the brake through the lever being supported by a small weighing machine known as the Denison balance. The total load or effective weight on the lever equals the lever weight less the load indicated by the balance above the lever. Example :—Let the effective weight = W lbs. ; the radius of the lever = 5 feet 3 inches ; revolutions per minute = N. The circumference of the circle of which 5 feet 3 inches is the radius is 33 feet, so that

$$\text{Brake Horse Power} = \frac{W \times 33 \times N}{33,000} = \frac{WN}{1,000}.$$

It will be noticed that by making the radius of such a length that $2\pi r = 33$, the equation is made very simple and the product $W \times N$ has only to be divided by 1,000 to obtain brake horse-power.

To set up the machine the turbine shaft is brought into line with the engine and coupled to it. Water is then turned on to the inlet pipe, air escaping from the outlet one. When the machine is in full action, the inlet valve is opened full, and the flow of water is regulated by the outlet valve, so as to keep up a certain amount of pressure in the casing. The water is retained in the casing by glands, the packing of which is in contact with the main shaft. Since the reaction against friction is in the same direction as that due to the water in the casing, no error is introduced by their employment. The quantity of water required to keep down the temperature is given by the formula

$$G = \frac{\text{B.H.P.} \times 33,000}{778\,(T_2 - T_1) \times 10}.$$

Where G is water in gallons per minute ;

T_1 inlet temperature in degrees Fahrenheit ;

T_2 outlet temperature.

This dynamometer possesses these qualities—namely, it can be applied to high-speed engines or motors, it absorbs very great power, while it can be regulated for any power within its range without the adjustment of weights.

The Peter Brotherhood Fluid Friction Dynamometer.

A hollow chamber, consisting of a shallow cylinder of which the diameter may be about twenty-two times the depth, is

carried on hollow trunnions ; these rest on antifriction rollers, carried by pillars on each side of the cylinder, which is called the casing (Fig. 54). Perforated annular plates are fixed to the casing, and between each pair a disc rotates, the discs being attached to a shaft which passes through the trunnions. The discs are close to the plates fixed to the casing, but do not touch them. When the discs are rotated frictional resistance is set up, which tends to rotate the casing. This resistance is

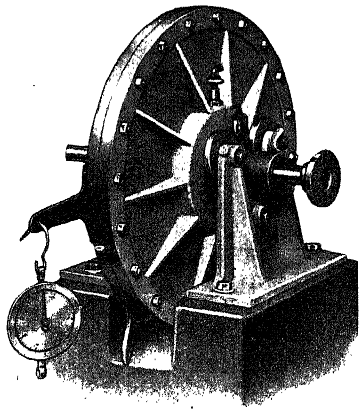


Fig. 54.

balanced by the moment of a force due to a spring balance, attached to a lever projecting from the casing. The energy imparted to the shaft and its discs is transformed into heat by the fluid friction, which heats the fluid. For a long test enough cold water must be supplied to the casing to keep the temperature down to a convenient limit. The load on the motor at any required speed is adjusted by varying the quantity of water in the casing ; and any desired quantity of water may be retained, and consequently any load may be steadily maintained, although water flows continuously through the casing.

CHAPTER VI

AIR BRAKES

THE invention of the air brake is due to Messrs. W. G. Walker & Co. It was patented by Mr. Walker on February 4,

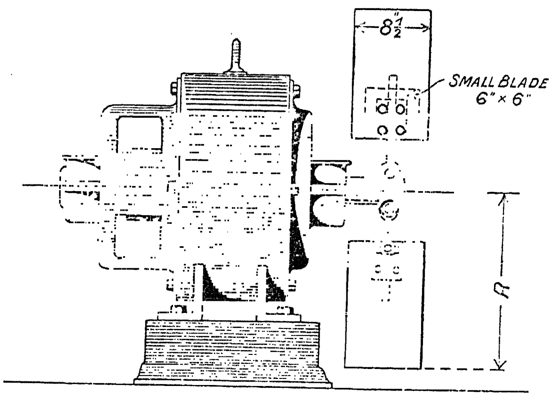


FIG. 55.

1904, and numbered 2,743. In this form of ergometer energy is absorbed by means of rotating vanes, which set air in motion (Fig. 55).

Two rectangular vanes are fixed on two radial arms, which are easily clamped on to the axle of the motor to be tested. Three different sized vanes are supplied with each ergometer. In order to make a test of a motor the ergometer is clamped on to the axle of the motor and the vanes are adjusted to such

a radial position that the motor runs at the required speed when under load. When the speed is known and the position and size of the vanes, the horse-power can be obtained from the calibrated results supplied with the ergometer.

[There is an illustrated account of a Walker dynamometer of extra large size in the *Engineer*, March 24th, 1911, p. 297. With this the power absorbed at 450 revolutions per minute with the plates in their extreme inward position is 65 horse power, while for the same speed at the extreme outward position it is 200 horse power. As the power absorbed varies as the cube of the speed, this machine at 1,000 revolutions per minute would absorb over 2,000 horse-power.]

[Col. Renard made for some years experiments with an air brake, which he called *Moulinet dynamométrique*, for the purpose of absorbing and measuring the power of high-speed engines and electric motors. An account of his experiments is given in the *Comptes Rendus de l'Académie des Sciences* for May 2, 1904, and these are referred to at length in the work "Dernier Evolution du Moteur a Gaz," by Prof. Aimé Witz, published by Louis Geisler, 1, Rue de Médécis, Paris, 1910. He employed a rectangular bar of ash which he could clamp crossways to the end of the shaft of the motor. The bar was divided from the middle both ways, and a pair of square aluminium plates were employed which could be secured to the bar in pairs at corresponding positions on either side of the axis. The plates were fixed so that the inner and outer edges of each were parallel to the axis of rotation and the plates were flat against the bar. He employed air brakes of different sizes according to the power required, and he established experimentally the following laws. With any particular combination of bar and plates the resistance was accurately proportional to the square of the speed of rotation or the horse-power was proportional to the cube of this speed and also to the density of the air. For air brakes similar in form but of different dimensions the horse-power absorbed at any speed of rotation was proportional to the fifth power of the linear dimensions. With different positions of the plates on the bars the constant is determined by experiment. The author speaks of the great convenience of this form of brake and of the very large range of power which can be obtained with brakes of very moderate

dimensions. He draws a very rigid limit for the speed, which must on no account be exceeded on account of the risk of accident.]

[An important investigation on the air brake has been made by Prof. W. Morgan and Mr. E. B. Wood, and a paper on the subject was read by them in June, 1913, before the Society of Automobile Engineers of New York. This is printed in the Proceedings of that society and a reprint will be found in the Proceedings of the Institution of Automobile Engineers (London) of 1914. The authors set out to investigate the laws governing the action of the air brake experimentally. For this purpose they employed a four-cylinder petrol engine to drive the brake, and in order to measure the power absorbed they mounted the engine and petrol tank like the motor of a cradle dynamometer, so that it could turn about the same axis as its own crank-shaft, being carried on large ball bearings for this purpose, and all balanced so as to be stable. The torque experienced by the balanced engine was measured as usual by means of a dead weight carried by an arm. The horse-power required to drive the fan at speeds varying in the ratio of over 2 to 1 or at powers of about 12 to 1 showed that the horse-power is accurately proportional to the cube of the speed, and this was found to be true with many sizes of plates at a number of different positions.

An investigation of a rational formula for the resistance, depending on the size and position of the plates, is given, but these do not lead to results which can be relied upon as the cube law of resistance may be, and a tabulated constant is in practice necessary. A very important part of the experimental investigation relates to the disturbing effect of walls or screens near the fan. A number of rings which together constitute a disc could be separately or together fixed near to and parallel to the plane of rotation of the fan on one side only. These reduced the power absorbed, so that the power calculated by the formula suited to a fan in free space was higher than the actual power by amounts varying from 5 to 21 per cent., according to the number of rings.

As screens were gradually built up close to and round the fan the excess of the calculated result became more and more, until when a rectangular box was built round the fan but open at the

top the formula based upon free access of air gave a result 194 per cent. too high, or nearly three times the correct amount. These show the great importance of allowing the fan to rotate in a clear space so that the air may circulate freely.

Tests were made with the barometer at different levels, but the range was very inadequate for this purpose, being only from 29·9 to 30·7 in. Within this range, however, consistent results were obtained, from which the variation of resistance was found to be twice as great as the variation of pressure, and hence of the density of the air. This would correspond with a law making the resistance proportional to the square of the density for which there is no theoretical justification, and it is inconsistent with the experience of Col. Renard. I do not think, therefore, that the evidence for such a law is sufficient, and where the air brake is used in circumstances leading to a considerable departure of the density of the air from that obtained under normal conditions, as at high altitudes, where the barometer is always low, or where a low barometer and a high temperature or the converse cause the density to be abnormal, a correction should be made depending on the new density. The following table may be useful for this purpose:

TABLE showing the number by which the power calculated from the constant of the air brake should be multiplied for different temperatures and pressures.

Barometer (Inches, Mercury).	Temperature (Fahrenheit).						
	0	20	40	60	80	100	120
25	1·060	1·107	1·154	1·200	1·246	1·293	1·340
26	1·020	1·066	1·111	1·154	1·198	1·242	1·288
27	·983	1·026	1·069	1·112	1·154	1·197	1·240
28	·947	·989	1·031	1·072	1·113	1·154	1·196
29	·914	·955	·995	1·035	1·075	1·115	1·155
30	·884	·923	·962	1·000	1·038	1·077	1·116
31	·856	·894	·931	·968	1·005	1·042	1·080
32	·829	·866	·902	·938	·974	1·010	1·046

Taking the density of air as 1 when the barometer is at 30 inches and the thermometer at 60° Fahrenheit, the reciprocal of

the density at other temperatures and pressures are tabulated. These figures will show to what extent the power calculated from the cube of the speed and the constant is in error. This calculated power should be corrected by multiplying it by the number in the table to obtain the true power, *i.e.*, if the standard conditions are those on which the constant of the air brake is based.]

[WHITE AND POPPE AIR BRAKE AND FORMULA.]

[In the *Automobile Engineer* for August, 1910, page 68, there is an article describing the White and Poppe testing systems.

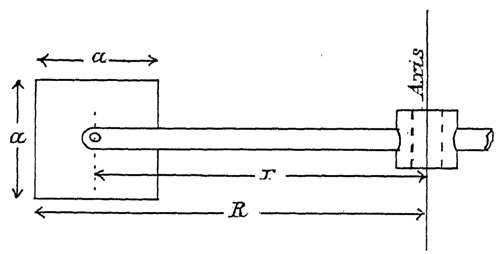


FIG. 56.

These engineers used the fan brake with square plates fixed with their centres at a distance r centimetres from the axis and with their outer edges $r + a/2 = \mathrm{R}$ centimetres from the axis as represented in Fig. 56, and, causing it to make N revolutions per minute, found as the result of experience that the horse-power could be obtained from the following equation :—

$$\text{Horse-power} = \frac{a^2 \times \mathrm{R}^3 \times \mathrm{N}^3}{4{,}010{,}000{,}000{,}000{,}000}.$$

They do not give this as a theoretically correct formula but one which for practical purposes is useful. As no allowance is made for the bar to which the plates are fastened, and as it clearly cannot apply accurately to plates of absurd dimensions, the reader must not attach too much importance to the formula or employ it with plates and radius bars of unusual proportions. As within its limits this seems to be a useful expression, I have

calculated and expressed on the accompanying logarithmic chart (Fig. 57) the horse-powers for a number of sizes of plates all set with their middle points at 55 centimetres from the axis. Each size of plate is represented by one of the eleven parallel

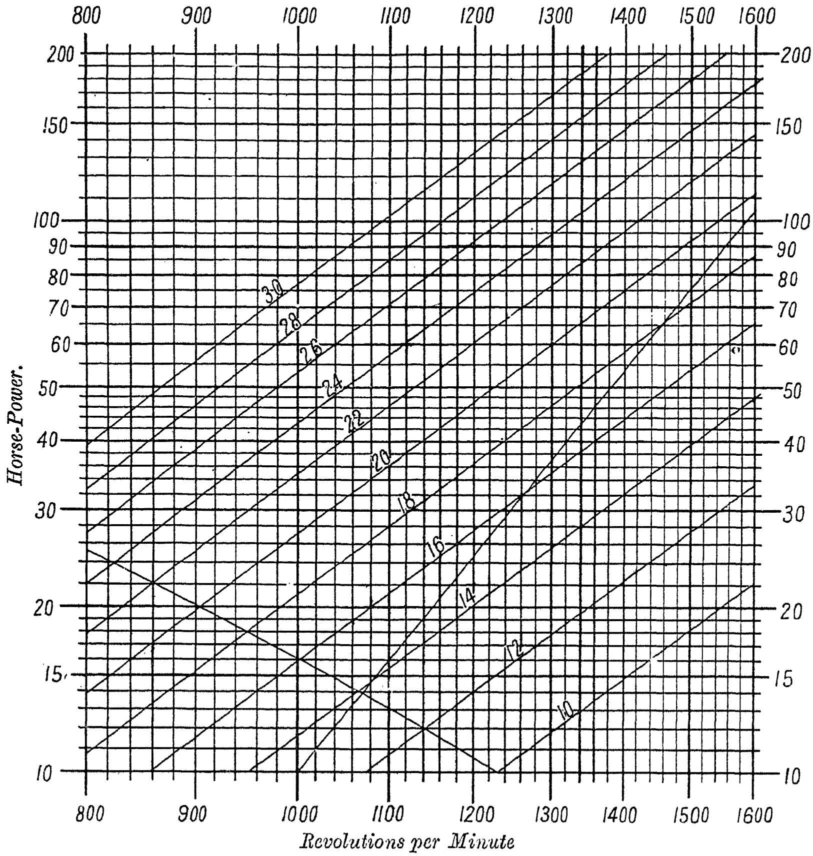


FIG. 57.

straight lines. Where these intersect the vertical or horizontal lines of the chart the horse-power and number of revolutions per minute, represented by the points of intersection on the vertical and horizontal scales respectively, are those that, according to the formula, should be absorbed by the particular plate. For instance, at 1,000 revolutions per minute the plate

20 centimetres square with its centre 55 centimetres from the axis should with the arm absorb 27·4 horse-power. As I have stated on page 26, the tangent of the angle which a line on the logarithmic chart must make with the horizontal to indicate the law $y = x^n$ is simply n; n in this case is 3. As, however, I have used a horizontal scale four times as great as the vertical scale, so that the useful range shall be contained in a chart of convenient size, the tangent of the angle is changed to $\frac{3}{4}$. There is a line sloping the other way (not quite straight, as the resistance does not, according to the formula, follow a law which is any exact power of the size of the plate), which I have called a scale line, * the purpose of which is this. If the horse power corresponding to a square plate of any other dimensions than those for which lines are ruled should be required, it is merely necessary to follow along the scale line until it intersects a horizontal line the numerical value of which is that of the side of the new plate. At the point of intersection draw a line parallel to the lines corresponding to the other plates, and this line will give the corresponding values for the new plate without any calculation. For instance, it cuts the sloping 10 line at 10, 12 line at 12, and so on.

The single line, drawn at a steeper angle than the others, is made to slope at an angle whose tangent is $\frac{5}{4}$. It therefore indicates a fifth power law, and it may be used to find the scale of magnification of an air brake necessary to increase the resistance to motion for all ratios up to 1 to $10\frac{1}{2}$ or 10 to 105.

If, then, the scale on which any actual air brake is constructed be called 1,000, then, following this line, it will be seen that as the scale is increased as indicated on the horizontal row of figures to 1,600 the resistance will increase in the ratio of 10 to 105 as read on the vertical row of figures, or a tenfold increase of resistance may be obtained by increasing the dimensions in the ratio 1,000 to 1,585, *i.e.*, so as to be very little more than half as big again in every dimension. As the fifth-power law is a true law, and is in no way dependent upon the empirical equation of White and Poppe, this line, or any line parallel to it, may be used with confidence.]

[The simplicity of the air brake and its law of resistance leading to great stability of speed make this type eminently

* *Nature*, July 18, 1895, p. 272.

suitable for testing motor-car engines; but it is necessary to bear in mind that the tabulated constants of the fan only apply accurately to definite conditions, and that where these are departed from, *i.e.*, where walls or partitions are so near as to interfere with the proper air movements, the rated power will be in excess of the true power. Care also should be taken so to use the air brake that it may not cause accident or nuisance. If an air brake is used at a speed beyond that for which it has been designed, especially if rigidly driven by a high-speed petrol engine with one or two cylinders, where the constantly repeated and severe stresses due to variations of speed are added to those already in excess of the proper capacity of the structure this may lead to accidents of a very disastrous character. The writer prefers at any time not to stand in the plane of rotation. The noise caused by the whirring of the plates is not of much consequence in an engine-house, but this may lead to trouble in a residential neighbourhood, especially if they move close to framing, for this gives rise to a beating of the air which is peculiarly disturbing. It is evident that this type of brake does not admit of adjustment of its resistance while running as the electrical and liquid brakes do.]

CHAPTER VII

MAGNETIC BRAKE DYNAMOMETERS.

IN 1824 Arago, a Frenchman great in physics, found that when a disc of copper was rotated under a magnetic needle supported on a needle point above it situated in the axis of rotation of the disc the magnetic needle followed the rotating disc in its direction of rotation after a certain speed was reached. The *rationale* of this experiment was given by Faraday,* who showed that the phenomenon of Arago was due to magneto-electric induction. Babbage and Herschel† investigated the matter and showed that the effect could only be produced with metals, while Arago held that the effect takes place with solids, liquids, and gases. Faraday was working on this phenomenon in order to discover the true *interaction* between the copper disc and the magnetic poles. He rotated a copper disc between the poles of a magnet, connecting the centre of the disc and the edge of the disc through a rubbing contact with a galvanometer. On rotating the disc Faraday found that a current was generated, and that the deflection of the needle of the galvanometer showed that when the direction of rotation was changed the direction of the current was also changed. Thus the first true dynamo was the outcome of this excellent experiment of Faraday. The peculiar feature of this prototype of the family of Dynamos is that the current is absolutely continuous, but from the nature of the arrangement the potential difference between the centre of the

* "Experimental Researches in Electricity," Vol. I.
† Phil. Trans., 1825, p. 467.

disc and its edge is very small. Foucault * devised an experiment by means of which the heating effect of internal currents (now called eddy currents) generated in a copper disc rotating in a magnetic field might be estimated, but Violle † was the first to estimate the *work* required to rotate the Foucault disc and thus heat it. This was a real ergometer experiment, made on the gravity method already mentioned. From this he deduced the heat equivalent as 435 kilogram-metres, a result too great when compared with the value 428 found by Joule.

This behaviour of a copper disc in a magnetic field is taken advantage of in the construction of galvanometers of the magnetic-needle type; the needle swinging over a copper disc induces currents in the copper which react on the needle and tend to bring it to rest; [also in electric motor meters the same interaction is utilised in order to obtain a resistance strictly proportional to the speed of rotation. Then, if the torque causing the disc to turn is made proportional either to the current strength or to the current energy, the rate of turning of the disc will be proportional to one or other, and the number of turns recorded on the dials will be a measure of the integrated current or energy, as the case may be.]

The Foucault phenomenon has been utilised by Messrs. Morris and Lister in the construction of an absorption ergometer, in which the eddy currents produced in two copper discs by a magnetic field generate a torque by their reaction in that field. This machine, which is called by the inventors an "Eddy current testing brake," is described thus by the inventors :—

"The Brake consists of two copper discs each mounted on a cast aluminium spider. These are made fast, one at either end of a sleeve which is keyed to the shaft of the motor under test (Figs. 58 and 59). Riding loose on this sleeve and between the two copper discs is an aluminium casting carrying a number of electromagnets, wound so as to have alternate polarity. These magnets consist of well-ventilated coils on circular cores fitted with pole pieces. To the outside of each copper disc is secured a ring of wrought iron, which revolves with the copper and at the same time forms a path for the magnetic flux. These

* Foucault, "Annales de Chimie et de Physique," 3me série (1855), T. XLV., p. 316; and "Recueil des Travaux scientifiques de Léon Foucault," p. 342, Paris, 1878.

† Violle, "An. de Chim. et de Phys.," 4me série T. XXI. (1870).

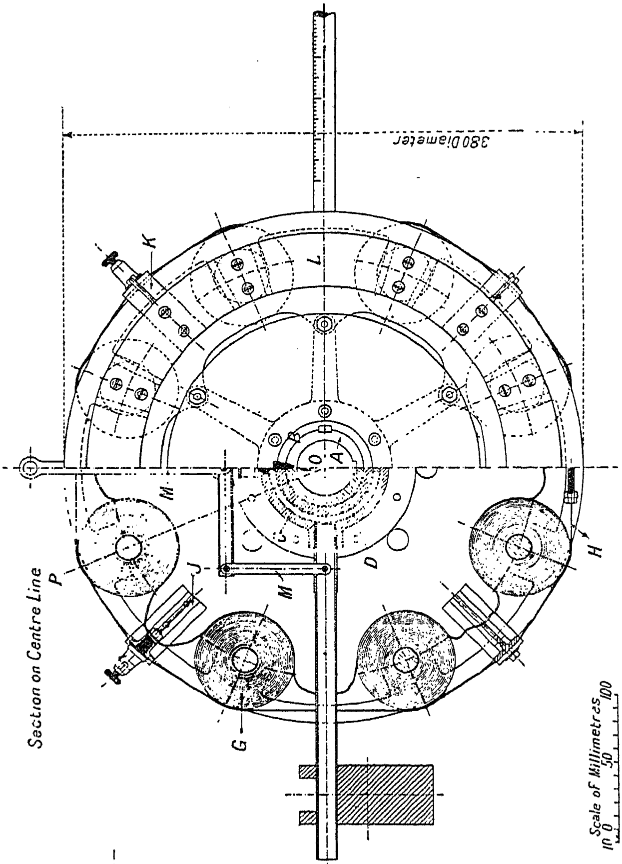


FIG. 59.

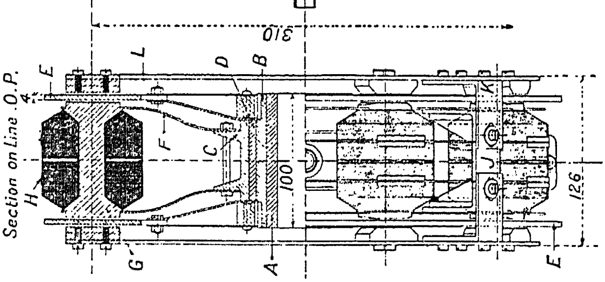


FIG. 58.

iron rings are fitted with cooling vanes, which dissipate the heat generated. When the motor is running the magnetic flux in traversing the moving copper induces eddy currents, which absorb the energy of the motor in heating the discs. At the same time the flux tends to drag

round the magnet system and levers. By suitably adjusting the exciting current the lever floats. The power is then given by—

$$\text{Brake horse-power} = \frac{\text{Torque in lb. foot units} \times \text{revolutions per minute}}{5{,}250}$$

"This force tending to turn the field magnets round is opposed by a gravitational force, due to a weighted lever. When these two forces are in equilibrium the lever floats between stops, in a horizontal position. The weight is so placed that when the lever drops below the horizontal position, the effective radial distance of the weight is reduced, and if the lever is above the horizontal the effective radius is increasea, but when the lever is horizontal, the weight acts at a definite measured length of lever to which it is adjusted. In the horizontal position of the lever the exact value of the moment of the weight about the axis is a known quantity."

CHAPTER VIII

END THRUST BRAKES

THE work-measuring machine of Bourdon (before 1870) is curious; its construction appears to differ entirely from that of any other work-measuring machine.

Two helical toothed wheels are in gear—one is driven by a belt by any motor, the other helical toothed wheel transmits the motion to the machine driven and under test. But, from the nature of the gear, the force acting in the plane of rotation has a component along the axis. Let the angle made between the slope of the teeth and the axis be β and P the pressure between the teeth in the plane of rotation, then the pressure P_1 tending to move the wheel along the shaft is $P_1 = P \tan \beta$.

This end pressure acts on a spring connected to a pointer, the deflection of which can be read on a dial. When the distance traversed by the point of application of the force is known per unit of time, and also the force, the power transmitted is at once determined.

The following description of an ergometer by the author is taken from a pamphlet on work-measuring machines (E. and F. N. Spon) published in 1884:—

"A torsion ergometer (Fig. 60). This form of ergometer is very different from any already described. It was used to control the motion of a dynamo worked by a windmill; the first windmill to which this arrangement was applied was nearly destroyed by the storms of September, 1882, at Taunton. Since then the plan of placing the dynamo in the head-cap of a windmill, thus avoiding the introduction of long shafts to bring down the motion, has been found to answer well; the conductors alone are brought down, the connection with the head-cap being made through rubbing contacts of copper on rings of the same metal. The wheel B is attached to the pulley A by means of two links

LL, as shown at KMN. The wheel A is fast on the shaft CD, and B is loose. If B be turned as shown by the arrow on the belt M the tendency of the links is to make the pulley B approach A and thereby compress the spiral spring S. A gun-metal wheel E, kept by means of a spring against the disc F, which is part of the pulley, moves a pointer over the dial H, and thus the tension of the belt at the effective radius is read. A speed indicator is attached as in the other machines. The central spring at the end where it comes in contact with the pulley B is furnished with a sleeve which slides on the central shaft. The end face of the sleeve is grooved, and the part of the pulley opposite to this groove is

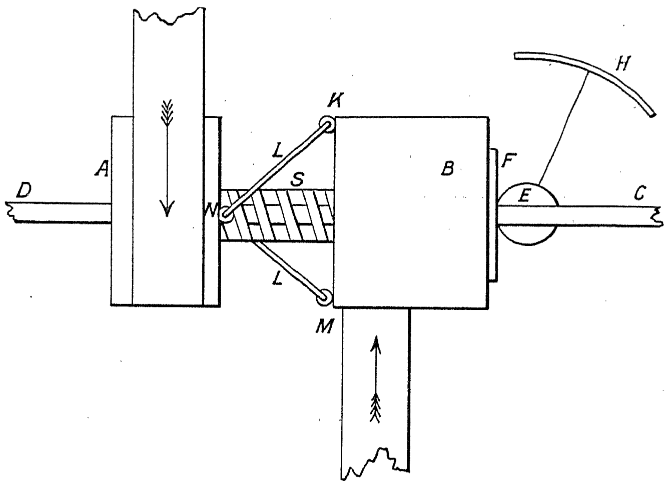


FIG. 60.

also grooved in a similar way. Several steel balls are placed between the grooves and render the contact between the spring and the pulley as frictionless as possible. The pulley B has a larger face than A, to permit of the slight side motion necessary to act on the spring."

In 1884 I devised and made an ergometer in which the tension of a belt was shown by the lateral shifting of one of two pulleys mounted on a shaft. The driven pulley was keyed to a shaft, while the loose pulley was connected to the fixed pulley through two rollers bearing on two spiral inclines which formed a part of the fixed pulley. The effect of increasing torsion between the two pulleys was to cause relative displacement and thereby compress a spiral spring. The axle of the machine was

tubular and the spiral spring pressed against a sleeve, connected through a slot in the axle with a cylindrical block within the axle; in this block a rod, projecting outside the axle, was free to rotate and actuate a pointer which indicated the difference of the tension of the belts on the two pulleys. The spring acted on the boss of the loose pulley, through a sleeve pressing against the boss through antifriction wheels. The loose pulley was wider over the face than the fixed one to allow for its small lateral displacement when running. The machine ran well and had practically no tendency to "hunt."

CHAPTER IX

HISTORICAL

[Memoires de l'Institut National des Sciences et Artes, Coulomb (Science, Math. et Phys.), T. II.]

In this memoir I have been principally engaged in determining to what degree a load more or less great is capable of diminishing the power (quantité d'action) which a man is able to yield in a day's work. The experiments which have been utilised on which to base that determination have been made in conformity with the most natural and common movements of men, such as walking horizontally, or ascending a flight of steps. The evident result appears to me to be that a man who ascends a flight of steps freely, and without any burden, is able to yield an amount of power nearly double that which the same man can yield when loaded with a weight of 68 kilograms, which is about the average load of men who carry up wood in houses. But since in that way of employing strength there is no useful work done besides the raising of the load, the result is that the useful work done by a man who ascends is not more than one-fourth of the total quantity of work done which a man yields in a day who ascends a flight of steps in the ordinary manner, and, allowing himself to fall, raises, by some means, a weight equal to his own weight. He will then produce nearly the same effect, or will do the same amount of work, as four men carrying similar weights on their backs. This observation appears to be of the greatest importance in guiding mechanicians in the construction of machines to be moved by men whose

power should be always employed in the most advantageous manner in producing useful effects. I further sought to compare the total quantity of power that men could yield in freely ascending a flight of steps with that which they gave when working in ringing a bell, or working a winch, etc., and I found that a man who ascended freely, *i.e.*, without a load, a flight of steps could do at least twice the amount of work that could be done in other modes of using their strength. The experiments which served to base the determination of the quantity of power used in the case of the bell and the winch were always made in the large workshops. I would ask those who may wish to repeat the experiments, if they have not time to measure the results after many days of continuous work, to observe the workman at different repetitions of their work during the day without the men knowing that they are watched. One cannot be too much warned of the risk of being deceived in calculating either the velocity or the effective period of work after a single observation of some minutes' duration. The results of all the preceding sections make the values for power much less than those made use of by the majority of authors in the estimation of machines; but the latter have been nearly all based on experiments which lasted some few minutes, and have been carried out by men chosen for the purpose; the calculations based on experiments, therefore, have been established on the supposition of effective work having been carried on for seven or eight hours per day. But in nearly every kind of work a man could put forth during some minutes an amount of power double or even treble his mean rate of doing work; he could even condense his whole day's work into two or three hours. This is what we have seen in the preceding section, where men who carry wood concentrate all their day's work into the time that they are subject to the load, and this is not more than one and a half hours during the day's work. The choice of men again greatly influences the determination of their mean strength. I have observed during ten years the carriage of earth moved by troops and by workmen, by the toise cube, as it is commonly called [toise = 6·39 feet]. I made fortnightly measurements, and found nearly always that the workmen belonging to the Grenadiers had gained by a third over the other companies, and often by a half over feeble

COMPARISON OF WORK DONE BY MAN (COULOMB).

Nature of work done.	Weight raised or stress brought into play.	Velocity or path traversed per second.	Work done per second.	Time during which work was done per day.	Quantity of work done per day's work.	Rate of doing work in h.p.
	lbs.	feet.	foot-pounds.	hours.	foot-pounds.	
A man ascending a gentle slope or steps, without a load, his work being that of raising his own weight.	143	0·49	70·5	8	2,030,400	0·128
A workman raising weights with a rope and pulley, the rope returning empty.	39·6	0·65	26	6	561,600	0·0473

workmen. If I had determined the mean strength of all the individuals who formed the workmen of the Grenadiers I should have found it a third greater than the mean strength of other workmen. It is true, and it must be remarked, that in this kind of work, of which the greater part consists in the wheeling of earth, not a single weak man is found amongst the workmen of the Grenadiers, and that two or three bad workmen amongst each of the other sets of workmen diminish the whole work done.

In conclusion, a variation of the mean quantity of power is due to food, but above all to climate. I have executed great works at Martinique carried on by troops—temperature 20 °C.—and I have also executed with soldiers the same kind of work in France, and I am sure that in this 14th degree of latitude, where the men are nearly always bathed with perspiration, the men are not capable of doing half the work that they can do in our climate, *i.e.*, in the climate of France.

Prony on Coulomb.

Results of many experiments devised to determine the power which men can yield in their work during a day, taking into account the various ways in which their strength is employed, by C. Coulomb.

Note on the memoir of Coulomb by Prony.

In order to give a clear and precise summary of this interesting memoir, it is necessary in the first place to determine the meaning of the words "*quantité d'action.*"

The effect which results from the mechanical work of men should always be reduced to raising of a heavy body; the velocity, moreover, with which this movement has been generated will die out if the cause which produced it shall cease to act, and it is necessary that a man should make a continual effort on the body to keep up that velocity. Here we have, then, two quantities which may be stated in numbers: the velocity, which is the number of metres or units of space traversed uniformly in the unit of time; and the stress (effort), which may be expressed and measured by a certain number of kilogrammes or units of weight. The product of these two

numbers represents and measures power (action), and when multiplied by the third number, viz., the time during which the stress acted, gives the quantity of power, or the total resultant effect of work, and it thus ranks as one of the things which may be measured and are susceptible of calculation.

Furnished with such information, the fundamental object of experiments is the comparison of the work done with the fatigue which necessarily follows it. The same quantity of work (or the number which expresses it) may result from an infinite number of different combinations of values of numbers, the product of which serves to measure it, combinations which depend on the different methods of utilising human strength. Is the fatigue, then, equal, in all cases, for the quantities involved for equal work, or does it vary under different circumstances, or does it cause the numbers which represent velocity, force, and time to vary ?

Daniel Bernoulli and other celebrated authors have adopted the former opinion ; but Coulomb showed that they had deceived themselves, and while he overthrew an error emphasised by such weighty authorities by means of proofs drawn from reasoning and experiment, he has done great service to applied mechanics.

Nevertheless, although fatigue may not be simply proportional to the quantity of work, it is a function of it—that is to say, the formula which represents it should comprehend, in some way, velocity, force, and time.

One knows, from the theory of mathematical analysis, that there must exist a certain relation amongst these three things such that a given effect may be produced with the least fatigue, or that with an equal fatigue the quantity of action or total effect may be a maximum. This is the problem which the author has proposed to solve, and which he has investigated in the different methods of utilising the exertion of man. He examined firstly the quantity of work that men were able to yield when they ascended, during a day of work, a slope or a flight of stairs with a load or without a load. The experiments which he cites on this point prove, to begin with, the inaccuracy of the opinion of Bernoulli ; he found that the quantity of work done by a man who ascended without a load, or who had only to raise his own weight, is double that of a man

loaded with 68 kilos. (each man working through a day), the weight of his own body being included. One sees then in a striking manner how, for equal fatigue and during a given time, that the total or absolute effect produced assumes different values through different combination of force and velocity.

But the word "effect" means here the whole quantity of work done in raising both the load and the weight of the man; and that which it is important to consider is the *useful* effect, that is to say, the whole effect, deduction being made for the value which represents the moving of the weight of the body of the man. This effect is the greatest possible when a man ascends without a load, but then the *useful* effect is *nil*; it is also *nil* if a man is burdened with a load so great that he can hardly move. There exists then, between these two limits, a value for the load such that the *useful* effect is the greatest possible. Coulomb supposed that the loss of the quantity of force is proportional to the load (an hypothesis which experience confirms). It furnishes an equation which, treated by the rules of maxima and minima, gives 53 kilos. as the load with which a man should be burdened, so that he may obtain the greatest *useful* effect in ascending a flight of stairs during a day's work.

And the quantity of *work* which results from this determination, and which amounts to 56 kilos. raised through the height of one kilometre, is sensibly the same as that given by experiment. But this method of doing work causes three-fourths of the total work to be lost, and consequently the cost of work under such conditions costs four times as much as when, in working, a man ascends a flight of stairs without any load, and then, allowing himself to fall, raises by some device a weight nearly equal to his own.

(The man descends on a seat attached to one end of a rope, which runs over a pulley, the other end being attached to the load.—F.J.J.S.)

"On the Flow of Fluids from Orifices in Vessels," by Monsieur le Chevalier de BORDA. [Mem. de l'Académie Royal, 1766.]

I again take an example from the theory of the resistance of fluids, of the bad use one may make of this principle. We

know that, in order to give a general solution of the problem of the resistance of fluids, one supposes a body D (Fig. 61) fixed in the midst of a fluid of indefinite extent, having a uniform motion in a straight line; we imagine, in addition, that the molecules of fluid, when approaching the body D, describe curved lines, *a b c d*, etc., *e f g h*, etc., or for the most part move in little curved canals *a b c d*, *e f g h*, etc., and one seeks to determine by the conditions of the problem the form of these little canals as much as the resultant pressure against the body D; but it is easy to see that each of these little canals necessarily has a portion *b f*, more narrow than the hinder parts *d h*, etc., which therefore may be classified with symptoms which we have mentioned in the preceding section; one need not employ in the case of this sort of movement the principle of the conservation of "*forces vives*"; but independently of the general proof there is one thing peculiar to the theory of the resistance of fluids; it is that in using, without restriction in that theory, the principle underlying it the result of the calculation will give a resistance equal to zero (*une resistance nulle*). To show this, suppose that the body D moves uniformly in a fluid at rest, drawn along by the action of the weight P: one knows, following the principle, the difference between the "*force vive*" of the fluid should be equal to the difference of the effective descent of the weight P: but when the movement has become uniform, the difference of the "*forces vives*" $= o$; therefore the difference of the effec-

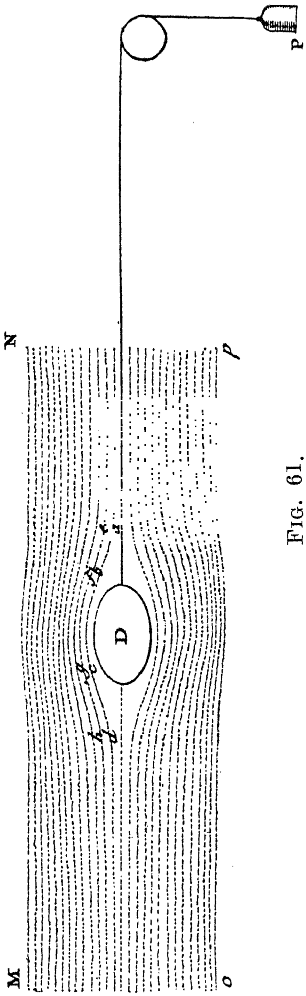


FIG. 61.

tive descent will $= o$. The weight then indicates the resistance of the fluid ; therefore the supposition of the principle which is at stake always gives a zero resistance.

"The Movement of Air Studied by Means of Chromophotography," by M. MAREY (Bulletin des Séances de la Soc. Française de Physique, Séance du 17 Janvier, 1902).

In this research on the movement of air M. Marey has made some beautiful and excellent experiments whereby the movements of air as it flows are clearly exhibited. The lines of flow of liquids such as water have been made known by the introduction of small streams of coloured matter, such as aniline dye. The method was employed by Prof. O. Reynolds, in his work on the flow of water through tubes (see page 142), and recently Prof. Hele Shaw has shown by means of streaks of coloured matter the lines of flow of a liquid, projected on a screen. M. Marey has dealt with the time measurements of the movements of a liquid by allowing the liquid, *e.g.*, water, to hold brilliant little bulbs of the same density as water in suspension, the bulbs being illuminated by bright sunlight. When obstacles of different forms are placed in water flowing by with the bulbs in suspension the lines of flow are at once evident. In one experiment the stream lines are shown as the water flows past a plane inclined to the original line of flow. The stream lines separate, some flowing to the right and some to the left, from a point which appears to be the centre of pressure. [Not only were the directions of flow made clear, but the speed at every point. With this object the illumination was made intermittent at the rate of ten flashes per second. The lines photographed were then beaded and the wider or closer spacing of the bright points in relation to a scale of millimetres gave the speed at every point.] M. Marey, going a step further, has produced some clear photographs of the flow lines of air. It would be nearly impossible to imitate the former method as used in the case of water and float minute balloons in air, but his excellent results have been obtained by introducing threads of smoke into air as it flows through a tubular chamber.

[The stream of air was drawn regularly through the experimental trough, and to ensure regularity was passed through two

frames over which silk gauze of extremely fine and regular texture had been stretched, one at the entrance and one at the exit of the trough. The smoke, which was produced by burning tinder, was led by a number of very thin and parallel tubes to the surface of the inlet gauze. The streams of smoke remained sharp and separate throughout the length of the trough. They were photographed by an instantaneous flash of magnesium light. When any obstacle such as an inclined plane or a ship-shaped body blunt at one end and sharp at the other was introduced the new lines of flow were clearly seen and the extent of the disturbance was shown to be greater, indicating greater resistance if the blunt end were at the down-stream end than it was if this were meeting the flow, in agreement with the forms of birds and fish and also of ships. When it was desired to obtain the speed at all parts of the stream lines the system of fine smoke tubes was kept vibrating by an electric trembler at ten vibrations per second. The lines then, whatever their form, had superposed upon this a fine ripple pattern, and the distance of consecutive waves in relation to a millimetre scale gave the velocity of the stream at all parts of the photograph.]

[The author included in his Table of Contents a reference to Prof. Hele Shaw's experiments on stream lines. These are described in the Transactions of the Institution of Naval Architects, Vols. XXXIX. and XL., the British Association Report, 1898, and in the Transactions of the Royal Society, Vol. CXCV., A.

Prof. Hele Shaw first made experiments on stream line motion past obstacles, rendering the motion visible by a froth of air bubbles included in the liquid. The air-bubble method was soon superseded by streaks of coloured liquid. The special feature of Prof. Hele Shaw's experiments was the use of an extremely thin lamina of fluid included between glass plates, and he used liquids differing so much in viscosity as water and glycerine. Sir George Stokes, who had seen Prof. Hele Shaw's results, showed (British Association Report, 1898) that a viscous liquid moving in an extremely thin sheet, though wholly different dynamically from a perfect or frictionless fluid moving in three-dimensional space, but with two-dimensional flow (obstacles being supposed in this case to

be indefinitely extended in a direction normal to the plane of flow), nevertheless had stream lines identical in form, so that experiments on viscous fluids in narrow channels could

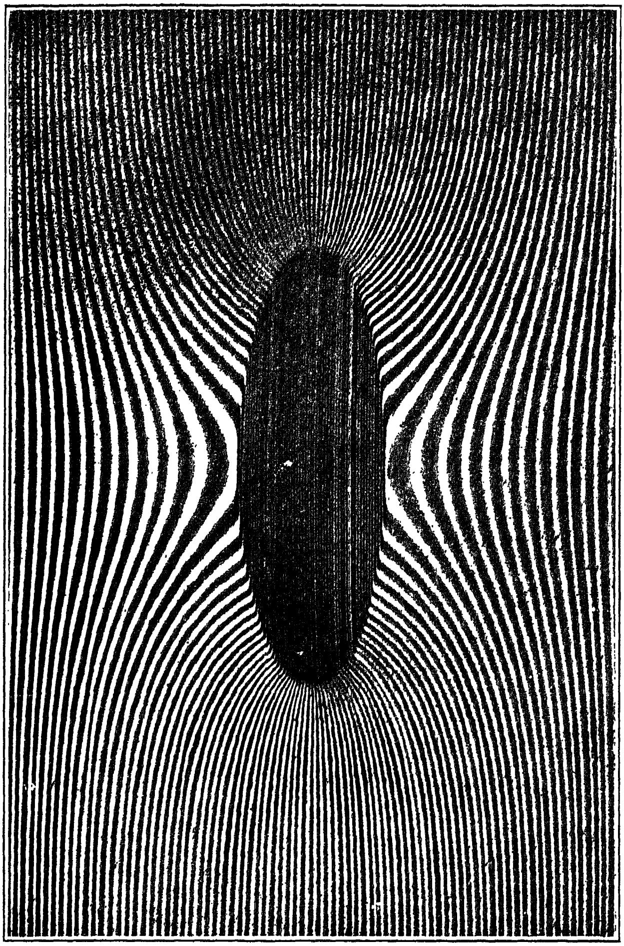


FIG. 62.

safely be used for ascertaining the forms of stream line flow of a frictionless fluid in two dimensions. Prof. Hele Shaw verified this experimentally in certain cases capable of being examined mathematically, and so was enabled to determine the forms

of stream line motion in other cases not susceptible of mathematical treatment. These experiments reached their highest development in the research described in the Royal Society

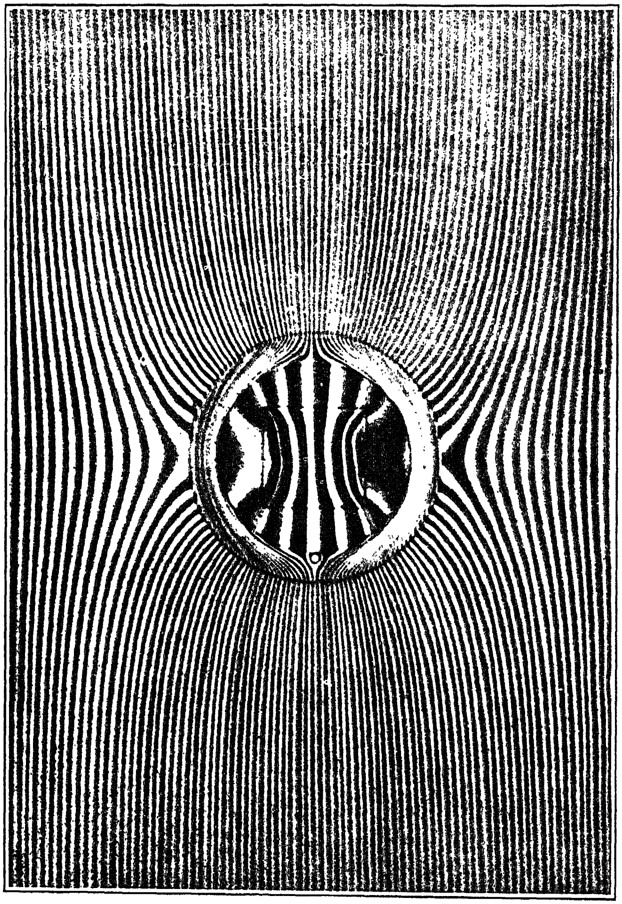


FIG. 63.

paper referred to above. The object is to find the solution of the forms of magnetic lines of force in a variety of cases. Here instead of an obstacle bodies of higher permeability than that of the surrounding medium are interposed. These allow magnetic induction to take place more freely through them and so are

imitated by the use of slides in which the narrow space occupied by the moving liquid has places of greater depth of the form of the more permeable body. The resistance to flow is inversely as the cube of the thickness of the layer, so the deeper portions represent greater permeability and any particular permeability may be imitated. The accuracy of this way of imitating permeability was verified by experiments with elliptical and circular depressions, the effects of which could be calculated, and so it was possible to get valuable information as to toothed armatures and other forms not amenable to mathematical treatment. It would be going too far from the subject of this book to give particulars of the singularly perfect arrangements made use of in this investigation, but as the results are so exquisite, and as they represent the highest perfection known to the writer yet reached in recording stream line motion, I am glad to be able with the permission of the Royal Society and of the author to reproduce two of the photographs. The first (Fig. 62) shows an ellipse of permeability 20 in a uniform magnetic field, while the second (Fig. 63) shows the screening effect of a hollow circular cylinder upon a square prism both of permeability 100 in a uniform magnetic field. It is hardly possible to believe that the dark and light bands are actually moving liquid and that they should contract, keep distinct, and widen again symmetrically as they do without losing their sharpness.]

[William Froude's Lecture.]

[The author left a reference to the interesting lecture delivered by William Froude, F.R.S., at the Royal Institution on May 12, 1876, and as this appears in his Table of Contents I have thought it desirable to give an indication of the subject treated. The true causes of the resistance of ships, depending on skin friction, eddy motion, and wave-making, were explained while the absence of any resistance due to obstruction equivalent to a pressure over the transverse section was shown in a series of propositions on stream lines, at least for an immersed body, to be wholly absent even though so plausible a theory had been universally accepted in past times.*

* De Borda's paper (p. 134) appears to have been overlooked.

(1) A plane surface moving edgeways through a frictionless liquid obviously meets with no resistance.

(2) A plane surface moving edgeways through a real liquid such as water meets with a resistance called "skin friction."

(3) A submerged body moving through a frictionless but heavy liquid should under the plausible ideas of the past meet with opposition, but the stream line principles to be indicated after the skeleton argument shows convincingly that there can be none.

(4) A submerged body moving through a real liquid is subject to skin friction, depending upon the extent of its surface and speed, such as might be obtained by drawing a sheet of the same total area edgeways at the same speed. It may also suffer from some small resistance due to eddies in the wake if the stern end is too blunt. There is no resistance due to obstruction acting over the cross-section.

(5) A floating body moving through and partly immersed in a frictionless liquid will, especially at higher speeds, generate waves the energy of which is derived from the motion of the body. This represents resistance.

The waves are formed because, as the stream line theory shows, the presence in the water fore and aft (at least in the case of ship-shape bodies) is above the normal, while it is below the normal amidships. The surface level of the water, therefore, is higher fore and aft and lower amidships. A wave travels at a definite speed, depending on and in proportion to the square root of its wave length. When the floating body approaches the speed at which the wave due to its motion also travels, the wave-making effect becomes greatly increased and the resistance rises rapidly. At much lower speeds wave-making is almost non-existent and the resistance is very small. If the floating body could be accompanied by a mechanism which would hold the water surface level and prevent the formation of waves, then the conditions of an immersed body would be met with and the resistance would vanish.

(6) A floating body moving through and partly immersed in a real liquid experiences a total resistance which is made up of all three—viz., skin friction, eddies in the wake, and wave-making. At low speeds the skin friction only is important;

the friction due to eddies in the wake is a small fraction of this. These two each increase in a rather higher proportion than the speed. The third kind of resistance as stated in (5) above is unimportant at low speeds, but may exceed the other two at high speeds. A longer ship may travel at a higher speed before the wave resistance begins its rapid growth than a shorter ship, but the skin resistance is greater.

The stream line theory can be indicated most clearly by considering the immersed body held at rest with the frictionless heavy liquid of great extent passing by it. As the liquid opens out and closes in again in its passage its movements may be mapped out by a lattice of imaginary tubes filling the whole space, curved in form and varying in section in such manner that the liquid in moving through these tubes should move along the paths and at the speeds that relatively to the ship it actually follows. The forms of these tubes in any particular case may be observed by introducing streaks of coloured liquid. The tubes will in the neighbourhood of the stem and stern of a supposed ship-shape body be wider than at a great distance, and they will certainly be curved so that the convexity is towards the body. The tubes in the neighbourhood of the middle parts will certainly be narrower, for the space occupied by the body is not available, and they will also be curved with their concavity towards the body. Considering first the effect of curvature, the liquid in all the different tubes in being deflected from the straight course will exert a pressure normal to the tube on its concave side which will be felt outside on its convex side. Thus, near the stem and stern the surface of the body will experience an excess of pressure which is the aggregate of that due to the liquid in all the imaginary tubes. Similarly, the middle parts will experience a corresponding diminution of pressure. Next, as regards the cross-section of the tubes, where the liquid is passing through a narrower portion it is flowing faster and its pressure is less, and conversely it is greater in wider portions. These changes of pressure in the liquid result simply from changes of its speed, and they are independent of the direction of motion. They depend upon the same hydro-dynamical principles that are made use of in the " Venturi " water meter.

The increase of pressure stem and stern and the diminution amidships, depending on the velocity of the liquid, must be added to the corresponding variations due to curvature of path in order to find the total increase at the ends of the body and diminution about its middle. It will be seen that these must be symmetrical and that the body is not subject to any force tending to make it follow the liquid. With a real liquid the only difference is the possible failure of the liquid to converge in true steam line motion and without eddies, and as the stern is made more abrupt the formation of eddies in the wake is made more pronounced. Whether the water moves past the body or the body moves through the water is immaterial, and so, except for the three kinds of resistance—skin friction, eddy friction and wave-making—there is no resistance due to movement through a liquid such as has been imagined and which may be considered as the pressure necessary to move the liquid out of the way acting over the immersed cross-section. The stream line theory shows that fine lines aft are the most important, and that a bluff bow has not the faults that would be anticipated. Nature has discovered this, and fish are a beautiful illustration.

I may add a reference to an investigation of great importance by Prof. Osborne Reynolds published in the Phil. Trans. Royal Society, Vol. CLXXIV., 1884, pp. 935—982, and in *Nature*, Vol. XXVIII., 1883, pp. 627—632. In this research Prof. Osborne Reynolds investigated by the aid of colour streaks the conditions in which continuous or parallel flow of water changed almost suddenly to discontinuous flow with a different law of resistance. The velocity that could be reached before this condition occurred depended upon the viscosity divided by the density of the liquid, and as in the case of water this is double at 5° C. what it is at 45° C. the critical velocity admitted of considerable range. This speed also became greater as the diameter of the tube was larger. For velocities below the critical velocity the thin colour band passed along the tube its whole length as a clearly defined line and the resistance was proportional to the velocity. When the critical velocity was reached the colour band almost suddenly formed whirls in the liquid and became uniformly diffused, but this never happened close to the entrance of the tube. The resistance then

appeared to vary as the velocity raised to the 1·722 power, not to the square of the velocity.

In the case of ships in air or water or aeroplanes speeds in which the surfaces in their passage would give rise to parallel flow, and hence to resistance proportional to velocity only, are too small to have any interest. Skin resistances at usual speeds follow a higher law than that of simple proportion.]

CHAPTER X

TRANSMISSION DYNAMOMETERS

Dynamometers of this type are placed between the prime-mover and the machine driven by it, and their function is to measure the power transmitted. Let it be supposed that the flywheel of a steam engine drives a dynamo, or any other machine, by means of a belt; then if we could make spring balances part of the belt, and the tensions of the tight and loose sides of the belt (often called the leading and trailing sides)

could be found and also the space through which the force acted, the work done would be known. Now, since this ideal condition cannot be realised, some machine must be employed capable of showing the difference of the tension of the leading and trailing sides of the belt, and also the distance through which this force has acted. Then the units of work expended during any instant equals the product of this force in pounds multiplied by the space in feet traversed in that instant. If by means of some mechanical process the differences of the tensions of the belt be continuously recorded and also the spaces traversed at each instant, during some known period of time, if the total time taken for any test be known, the data recorded afford a means of finding the average horse-power absorbed by the machine driven by the engine.

In the history of power-measuring machines M. le Général Arthur Morin * occupies the position of being the originator of dynamometric methods of measurements of peculiar and lasting excellence. He made his dynamometer self-registering, and exhibited the product of Force multiplied by the Space through which the force acted as an Area. He also invented and added to the dynamometer an integrator, by means of which the value of the area generated was found and the whole work done during any period of time estimated. As his work is so important, I here give nearly in full a free translation from the French of his description of his machines and instruments with exact copies of the original diagrams.

General and particular conditions which dynamometers and apparatus destined to measure work developed by animate or inanimate sources of power should satisfy.—It has been already shown that the work developed by a constant force F which traversed a path E, with its point of application in the desired direction, was measured by the product FE ; if the force had been variable, the total work developed when it had traversed any path E would have been the sum of all the elementary quantities of work, such as Fe, successively developed through the elements e of the path traversed. In the last case it has been shown how, by the help of the calculus, or the method of quadrature of Simpson, the sum of products such as Fe has been found for the whole given path E traversed in the direction

* "Notions Fundamentales de Mécanique," A. Morin, Paris, 1855.

in which the force acted. Finally the mean force of a variable one has been defined, and it has been shown how one may deduce the whole force by dividing the whole work by the whole path traversed. Apparatus destined to measure work developed by motors should indicate the product of the acting force and the path traversed, whatever their simultaneous variations may be. The illustrious Watt is the first who has satisfied the conditions mentioned in the construction of force measuring apparatus, to which he gave the name Indicator of Pressure. These are as follows :—

(1) The sensitiveness of the apparatus should be proportioned to the intensity of the forces to be measured, and should not alter through the apparatus being used.

(2) The indications of the bending of the spring should be obtained without calling for attention, or the inclination or bias of the observer, and should therefore be furnished by the apparatus itself, by means of traces, or material results which are left after an experiment is finished.

(3) It is necessary that the force brought into play at each point of the space traversed by its point of application should be found, or in certain cases at each instant that the observation lasted.

(4) If the experiment be extended over a long time, it is necessary that the apparatus should provide for the totalisation of the quantity of work given out by the motor (*i.e.*, an engine of some kind). In order to satisfy condition (1) springs must be employed which bend in proportion to the forces acting, and which have the shape of bodies of equal resistance. This produces greater facility for recovery, and gives to the apparatus great sensitiveness.

Rules for finding the proportions of the blades of a spring.— The theory of the resistance of material to bending, in agreement with the known results of experiment, shows that when a metal lamina of constant rectangular section is held in a recess by one of its ends the deflection varies directly as the load P and the cube of the length of the blade C, and inversely as the width of the blade A, the cube of the depth of the blade B, and the modulus of elasticity E.

If the longitudinal profile (in depth) of the lamina is parabolic, for bodies of equal resistance the deflections under the same load

are double those which a lamina uniform throughout its length would give while its resistance to breaking remains the same. For springs of equal resistance we have $F = \frac{PC^3}{EAB^3}$, a formula by the aid of which one is able to calculate any of the quantities which enter into it when the rest are known. Experience in the construction of a great number of laminæ for springs has shown that when made of German steel of good quality, hardened and tempered to a suitable degree, the value of the coefficient of elasticity to be used was $E = 20,859,000,000$. This is estimated per square metre, it equals 2,085,900 kilograms per square centimetre, or 29,668,000 pounds per square inch (kilograms per square centimetre multiplied by 14·2262 = pounds per square inch).

The relationship which it is convenient to establish between the different proportions of a spring.—The width a of the lamina should be limited to 4 or 5 centimetres at the most, because distortion caused by hardening is more marked as the lamina increases in size. It is this which introduces difficulties in adjustment. The observations made by me on springs have shown that the deflections of springs remain proportional to the applied forces, providing they do not exceed one-tenth of their length, for the strongest and one-ninth for the weakest, the measure being taken outside the fixed end. Accepting these data, it will be easy to calculate the thickness (depth) B which it will be suitable to give to a lamina at the fixed end, so that under a certain load it will take a known deflection. This dimension is given by the formula

$$B^3 = \frac{PC^3}{EAF}.$$

Longitudinal profile of the lamina of a spring.—The form of the longitudinal profile of the spring was deduced from the formula $y = \frac{B}{C}x$, the values of B and C being those already given and the origin at the external end of the lamina.

Arrangement of the blades of the springs.—The laminæ of the springs used to measure the traction of carriages, ploughs, boats, etc., are shown in plan and elevation (Figs. 64 and 65). Two laminæ, aa^1, bb^1, exactly similar, having their inner faces

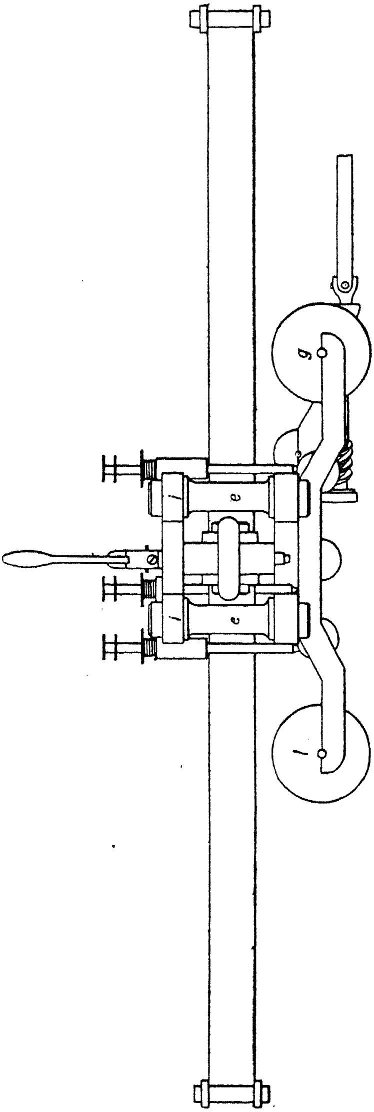


FIG. 64.

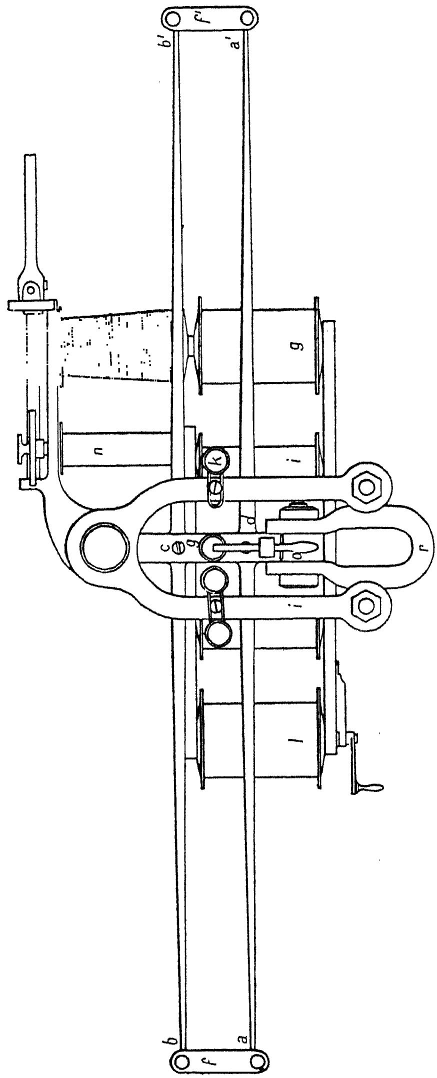


FIG. 65.

plane and their external ones parabolic, are terminated at their ends by connecting hinges of the same width as the laminæ, bored with a hole. Small steel bolts fit these holes easily and engage with links *ff*, to which they are fixed with nuts. The

lower shackle *c* was pierced with a recess to take the blade or lamina which was introduced lengthways ; a shoulder, of the length of the shackle, was fitted to the middle of the lamina and entered the recess exactly. Set screws *g* with conical points held the lamina in position. An external upper shackle *d* engaged the lamina *aa* and was furnished with a ring *r*, to which was attached the splinter bar or the rope on which the motor pulls. In order to measure large forces, four laminæ were employed, the resistances of which united to balance the forces present. Deformation of the laminæ was prevented by fixing in the shackle *c* two stops *i* joined by two crossbars *e*, against which the outer lamina came in contact when the tension exceeded its highest limit.

Arrangement for obtaining a permanent trace of the deflections of the spring.—The front shackle carried a screw in a slot, by means of which a copper tube, terminated by a conical socket in which is fitted a quill pen, could slide and be retained by friction. The tube was filled with Chinese ink of suitable consistence. When the pen was well wetted and held correctly in its conical socket, capillarity sufficed to feed it constantly and regularly. The pen might be replaced by an ordinary lead pencil, or by one that does not require sharpening, but then a pressure of about 40 grammes would be required to make a sufficiently visible trace. The traces of the style were made on a band of paper coiled on a cylinder *l*, which served as a magazine ; the paper band passed over three small cylinders, which guided it under the styles and prevented its being bent by the wind or by its own weight. The band of paper was coiled on to another cylinder *g*, which acted as a receptacle for it and on to which one of its extremities was fixed with gum. A second style *k*, carried by one of the check pieces, and therefore immovable, traced on the paper a line which corresponded to no force, or the position of the laminæ when at rest, and it gave thus the zero of forces, so that the force acting is always measured by the displacement of the curve described by the moving style from the zero line.

The method of moving the paper which received the trace of the style.—The motion of translation at right angles to the direction of the forces acting was transmitted to the band of paper by means of an endless cord which passed over the nave of one of

the wheels of the carriage and over a return pulley. On the prolongation of the axle of this pulley was an endless screw parallel to the laminæ, which engaged with a pinion fixed on the axle of a small cylinder. On this was coiled a silk cord which transmitted the motion to the cylinder which received the paper band. By suitably proportioning this transmission gear with bands of paper 16 to 18 metres long, one could (using only one band) extend experiments over 800 to 1,000 metres or more. If the movement was transmitted directly to the axle of the receiving cylinder, the diameter of which was increased by the paper as it rolled on to it, the translation of the paper was accelerated. In order to prevent this inconvenience, the silk cord was coiled on a small intermediate cylinder fixed at its free end to a conical fusee having a screw

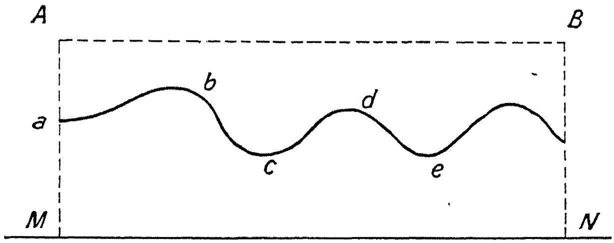


FIG. 66.

cut on its surface, its diameters being calculated, so as to compensate for the gradual increase of diameter of the receiving cylinder.

Observations on the quadrature of curves traced.—From this description it is seen that the paper unrolls under the style with a speed which has a constant ratio to that with which the road was traversed; lengths of paper band represented the length of road on a scale known from the ratio. The ordinates of the curve of deflections measured from the zero line were proportional to the forces acting; the result then was that the area included between the curve, the zero line, and any two ordinates, represented the total work done in that interval by the prime-mover.

Methods of finding the quadrature.—Tedious methods are mentioned, and dismissed. The first method, which requires no calculation, consists in drawing a line AB (Fig. 66)

parallel to the zero line MN, at a given distance from it, greater than the maximum deflection, or equal to it. A constant fictitious force will correspond to that ordinate, to which will be due a known amount of work represented by the area of the rectangle MNBA. But *abcd*... NM was the real curve of forces given by experiment. The following ratios are therefore evident :—

$$\frac{\text{Area MNBA}}{\text{Area } abcd \ldots \text{NM}} = \frac{\text{Work due to constant fictitious force}}{\text{Work sought.}}$$

Since the paper was machine-made of uniform thickness, so that the areas were to one another as their weights, by cutting them out and weighing the entire rectangle, and then the area bounded by the curve, the work could be found by simple proportion. Example :—When a 700-kilogram spring was used, 1·25 millimetre corresponded to a force of 10 kilograms, and a constant deflection, or a height of rectangle equal to 70 millimetres, corresponded to 560 kilograms. Calling P the weight of the band, 70 millimetres high, and p the weight of the part bounded by the curve and the zero line, E the length of the road traversed, F the mean force developed by the motor, we have

$$F = 560 \frac{p}{P} \text{ kilograms,}$$

and the whole work done by the variable force will equal the product EF.

Use of the planimeter.—The second method of obtaining the quadrature of the curve quickly without calculation is by the employment of the planimeter of Ernst (Figs. 67 and 68), which is furnished with a cone made of wood. This instrument consists of a cone *bcb*, the axis of which is inclined with respect to the plane of the table which carried the instrument, so that (looking at a vertical section of the cone) its uppermost edge is parallel with this plane. The cone is carried on points by two supports fixed to the frame XX, and on the prolonged axle there is a small roller *aa*, which presses against a strip LL parallel to the guides, directed by which the frame XX moved. When the frame was pushed in either direction along LL, the roller and cone rotated and made a number of turns proportional to the path traversed by the frame. A counter, of which the most important organ is a roller *dd*, having its plane

of rotation vertical and perpendicular to the upper horizontal edge of the cone, turning about an axis parallel to that same

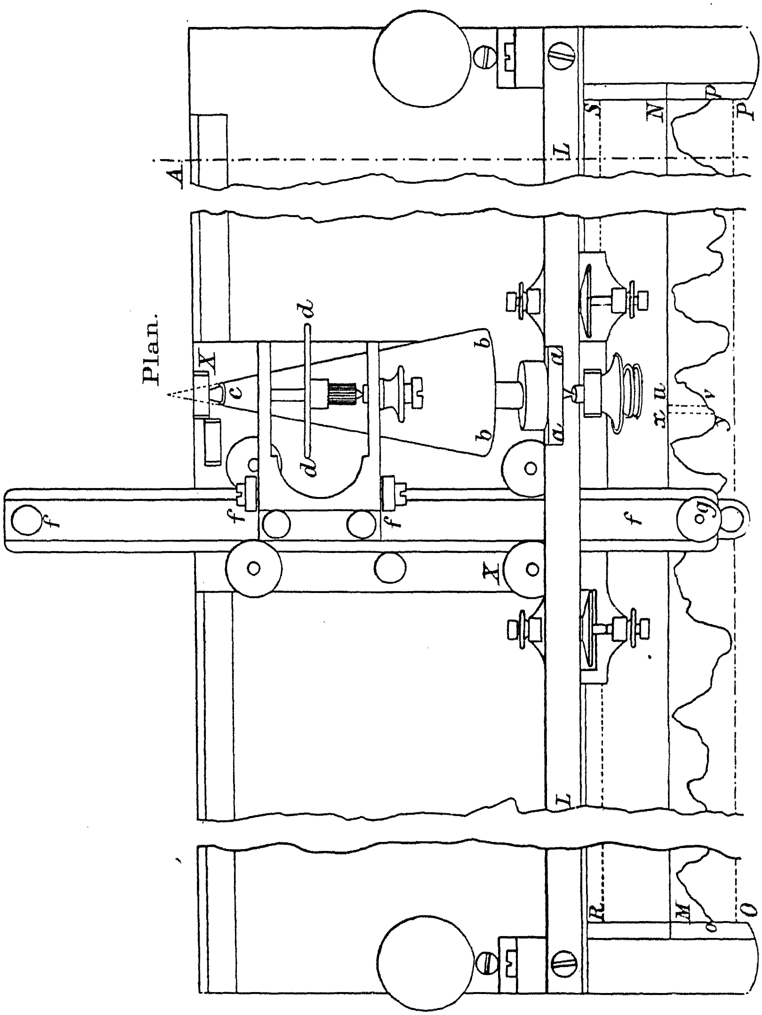


FIG. 67.

edge, is carried by a U-shaped bearing which forms a part of the transverse slide *ff*, which moves with the frame XX, and is also capable of motion in a direction at right angles to the strip LL, so that the roller can approach or recede from the

vertex of the cone as desired. The counter rests on the surface of the cone by virtue of its own weight, and when the cone revolves the roller does so also, and it is evident that the number of revolutions it makes is proportional (1) to the number of revolutions of the cone, or the length of the path passed over in the direction LL, and (2) to the distance of the roller from the vertex of the cone, or to the product of these two quantities. This being so, let us suppose that the roller was at the vertex

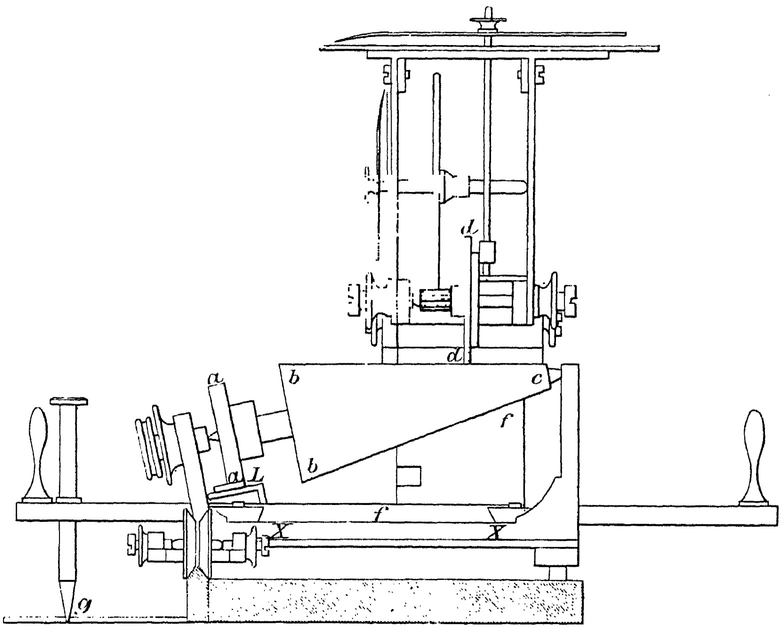


FIG. 68.

of the cone. A point g on the slide ff corresponds to a line RS parallel to the guide LL : let it be on R ; it is evident that if the frame XX is pushed so that the point follows exactly the line RS, the roller will not rotate, since the velocity of the vertex of the cone is zero, but if the point g is on M, and the roller is distant from the vertex of the cone by a quantity equal to MR = NS, when the point is moved from M to N, the number of turns of the roller will be proportional to the length RS, which is the base of the rectangle MNSR and to the height of the same rectangle, and therefore to the area of

this rectangle. In the same way, if the point *g* follows the line OP, the number of turns of the roller will be proportional to the rectangle ORSP. In the construction of the instrument the roller need not reach the vertex of the cone, and therefore it is left out and the method of finding the area of the rectangle is slightly modified. Suppose, for example, one wishes to estimate the area of OMNP. The point *g* is brought on to the line MN, care being taken that it remains on the line during the traverse of the frame XX. The whole instrument is then pushed so that the point *g* passes from M to N. The roller of the counter makes a number of rotations proportional to the rectangle RMNS. The point *g* is then brought over P, then the frame is pushed backwards, so that the point *g* follows the line PO. During this retrograde motion the roller revolves in an opposite sense, and makes a number of turns proportional to the area of the rectangle ORSP, and since in these two consecutive movements it has rotated in opposite directions, it is evident that the final number of turns made is proportional to the difference of the two rectangles ORSP and MRSN, or to the rectangle OMNP. The movement of the roller is transmitted by gear to two pointers and dials, one of which shows units, tens, and hundreds of square millimetres, and the other thousands of square millimetres. What has been said about a rectangle applies also exactly to the quadrature of a bounded surface, such as those traced by the styles of dynamometers, bounded on one side by a straight line and on the other by an undulating curve *op*, since each element *uvyx* of that area may be regarded as a little rectangle of which the base is *ux*, and its height the arithmetical mean between *uv* and *xy*. In order to make the reduction of the curve or the quadrature of the area MN*po*, one proceeds as follows. The sheet of paper is fixed under the table of the planimeter, so that the point *g* moves as near as possible to the table; it follows accurately the line MN of zero force when the frame XX is pushed from M to N. Then the point *g* is brought on to M, the counter is lifted, and the two pointers set to their zeros; the roller is gently placed on the cone, and the frame XX pushed so that the point *g* moves from M to N. The slide *ff* is moved so as to bring the point *g* on to *p*; then by means of the compound movement imposed on it the point follows the

bends of the curve until it arrives at O. The dials are then read, and they show the number of square millimetres contained in the area operated on; dividing this number by the length of the base MN, measured in millimetres, the quotient gives the mean ordinate, or height of the rectangle equal to the same area, and therefore the mean force which has acted. In order that the operations described shall give an exact result it is necessary that in the motions either forwards or backwards the roller shall have no slip while revolving. This condition of not slipping is obtained by employing a cone of unpolished wood instead of the polished metal cone of ordinary planimeters.

Dynamometer for totalising the work done during an interval of time or over a long road.—When the work done by motors during their passage over a long road is sought, the dynamometer furnished with the style and band of paper is inconvenient, and it is important that an apparatus which itself gives the total of the successive elements of work, so as to dispense with the quadrates described, should be employed. Such is the object of the following modification introduced in the dynamometer mentioned in the preceding paragraphs. The back of the shackle is traversed by an axle rotating in it, on which is screwed a disc B (Fig. 69) having a diameter of 16 centimetres. Below the springs, on the axle of the disc, a pulley is fixed, which is rotated from the wheels by means of an endless cord passing round return pulleys. A column E, which forms a part of the shackle *d*, supports a counter which necessarily follows all the movements of deflection of the outer lamina of the spring. The important organ of the counter is a little roller carried on an axle parallel with the disc, its axis being in the line of traction. This roller acts in the same manner as that employed in the planimeter only; since in place of a

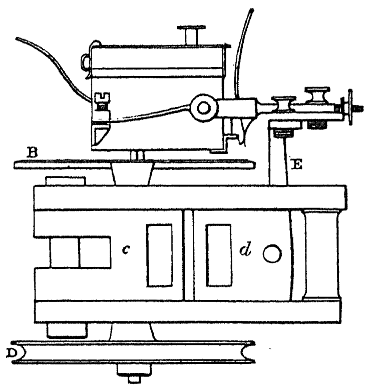


FIG. 69.

cone a disc is employed, the roller is able to reach the centre of this circle when the instrument is at rest. After what has been said respecting the planimeter it is not necessary to explain the action of this apparatus, and we know that the number of turns of the roller is proportional to the sum of the elementary products of the forces acting and the elements of the path traversed, or to the whole work done. The radial distance of the roller from the centre of the disc is reckoned in metres, under the strain due to the traction F, expressed in kilogrammes; the radial distance is the deflection of the spring under this stress, providing that the apparatus is so arranged that the roller rests at the centre of the disc when there is no stress. Let

r_1 = radius of the roller;

e = the path traversed in one second by the carriage in the direction in which it is drawn, when the pull is constant, or in an infinitely small time, when the pull is variable;

R = the radius of the wheel whereby it is moved;

$n = \frac{e}{2\pi R}$ = the number of turns of the wheel corresponding to the length of the path e;

$K = \frac{F}{r}$ = the relationship of forces to measured deflections;

N = the number of turns of the roller for a path e;

R′ = the radius of the nave of the wheel from which the movement of the disc is derived;

r' = the radius of the pulley of the disc.

It is evident that the disc makes a number of revolutions $= \frac{R'}{r'}$ for one turn of the wheel, or $\frac{e}{2\pi R} \frac{R'}{r'}$ for the path e traversed in the direction in which the carriage is pulled. The roller will make $\frac{r}{r_1}$ turns for one turn of the disc; we shall have then $N = \frac{e}{2\pi R} \frac{R'}{r'} \frac{r}{r_1}$ for the number of turns of the roller corresponding to a path traversed $= e$ under a pull of traction F. The number N was moreover finite or infinitely small, provided that a constant force was dealt with, or a variable

force and an element of the path. We have by the definitions

$$K = \frac{F}{r} \text{ or } r = \frac{F}{K},$$

and therefore
$$N = \frac{R'}{2\pi R r' r_1 K} Fe;$$

or
$$Fe = \frac{2\pi R r' r_1 K}{R'} N.$$

Thus, either in the case of a constant force and a finite path, or a variable force and an elementary path, we see that the work developed by the motor is measured by the product of the constant factor $\frac{2\pi R r' r_1 K,}{R'}$ and the number of turns N or the elementary fraction of a turn made by the roller; provided that the whole work at the end of any interval was the sum of elementary quantities successively developed, it will be equal to the same product by taking N equal to the turns of the roller during the observed interval.

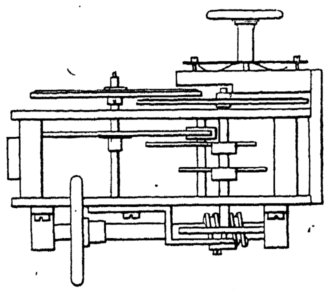


FIG. 70.

Apparatus of this kind has been employed with success and ease in prolonged experiments on the draught of carriages, and afforded means for determining the total work done by teams of horses six in number during a whole day's work on the roads connecting Paris and Amiens and Nancy and Mans.

Arrangement for indicating the number of turns made by the roller.—It is easy to see that the axle of the roller furnished with an endless screw (Fig. 70) may be made to communicate its motion by means of gear to two dials, one of which shows units and tens of turns, and the other hundreds and thousands of turns of the roller. In order to observe the divisions of the dials without stopping the apparatus or the progress of the carriage, two styles fed with thick ink are arranged so that they mark the enamelled dials when a button is pressed with the finger. Observations can thus be made and repeated without any confusion in the results.

Chronometric motor dynamometer.—When experiments are

required on the resistance of towing boats or vehicles without a forecarriage it is difficult, and in some cases impossible, to move the band of paper with a speed varying exactly as the path traversed. In this case it is much more convenient to employ a chronometric motor for giving the paper a sensibly uniform motion. Then lengths of paper generated in an experiment represent times, and the quadrature of the curve of deflections gives the sum of products such as Ft, viz., of each force, by the elementary time of its duration, or that which we shall call, as will be seen later on, the total quantity of motion developed in the interval of time under consideration. By dividing the area found by the whole time, or by the length of the paper generated, we find the mean force of the motive power. In the haulage of ships, and in all cases in which speed would modify the results, two additional pencils are required; of these one serves to mark points on the paper corresponding to equal intervals of time, viz., fifteen or thirty seconds, and the other distances traversed between posts or objects at a known distance apart.

Rotational dynamometers.—The apparatus which has been described was constructed with a view to measure the power developed by motors, the direction of the action of which was in a straight line, or a circular path, but it has been easy to modify the machine so as to render it suitable for finding the work transmitted by a rotating shaft to any machine by employing the principle either of the styles or the counter.

Description of the rotating dynamometer with the styles and paper recording apparatus.—On a shaft carried on two cast-iron columns (Figs. 71 and 72) three pulleys of equal diameter are placed; of these A is fixed to the shaft, the other, C, next to the first, is an idle pulley, and the last, B, is movable on the shaft between limits which will be shown. This dynamometer was placed between the shaft of the motor (the engine) and the machine the resistance of which was sought. The idle pulley was embraced by a belt driven by the motor. When it was shifted on to the pulley A the shaft revolved, with a velocity which depended on the relationship between the diameter of this pulley and that of the pulley of the motor. The pulley B was furnished with a belt which transmitted the motion to the machine under trial and overcame its resistance; but,

since this pulley is loose on the shaft, it would not be carried round by the motion imparted to the shaft by the fixed pulley, unless a stop which forms a part of it were pressed by the end of the radial lamina of the spring which is fixed in a boss. This lamina, turning with the boss, acts on a stop, the resistance of which causes it to bend, and when its resistance to bending

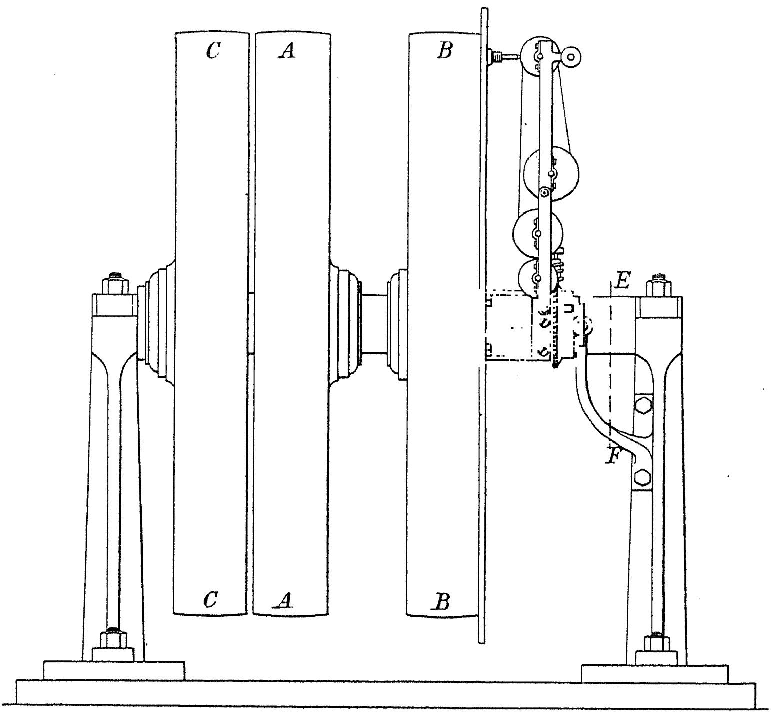


Fig. 71.

is sufficient to overcome that of the driven machine, motion begins, and is transmitted to the shaft of the machine under trial, by the intermediary spring, the deflections of which are a measure of the resistance to be overcome. A style attached to one of the spokes of the pulley touches a band of paper moving at a speed proportional to the speed of the shaft, and on it it traces a curve of deflections in just the same way as in the dynamometers used in testing carriages. Another style, which however is fixed, traces at the same time a line

corresponding to no deflection, or the position which the moving style occupies when the force is zero. This zero line is found about the middle of the width of the paper band, so that the force can be measured indifferently in one sense or the reverse. The laminæ are of parabolic section, and one can

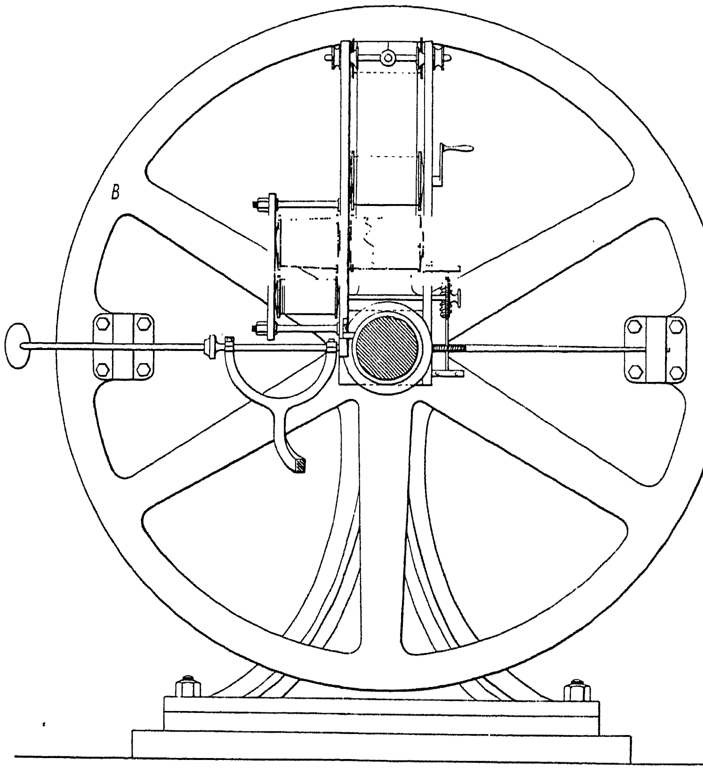


Fig. 72.

employ as many as one wishes, according to the intensity of the forces which the apparatus is required to measure. A fixed stop on the shaft limits the displacement of the pulley, and therefore the deflection of the spring; this prevents overloading in case of accidents.

The transmission of motion of the shaft to the band of paper.—A collar provided with helicoidal teeth rides loose on the shaft; its teeth engage with a pinion the axle of which (viewed

in a plan perpendicular to that of the shaft) does not meet this. The axle of this pinion is furnished with an endless screw, which engages with another pinion carried on the prolongation of the axle of the small cylinder on which the silk cord is coiled which moves the fusee. To start the band of paper the toothed collar is stopped by means of a clutch, against which a stop projecting from the collar comes in contact. Then when the toothed collar is fixed in space, so that the pinion carried by the shaft rolls over it, this pinion receives a relative motion which it transmits to the screw, the fusee, and the band of paper.

In apparatus of this kind a conical fusee serves to regulate the movement of the bobbin which carries the paper ; by the introduction of this intermediate organ the growth of the diameter of the paper on the receiving cylinder is compensated for, as has been already shown.

The rod which moved the roller of the integrator also actuated a tracing point, which marked a curve on a band of paper moving at a rate proportional to belt speed. The mean force acting was found from the area of the curve, and also the whole *Work* done. In a dynamometer built for the delivery of 50 horse-power the springs were six in number, made of steel of flat-tapered shape, fixed to the central boss, and bearing on rollers at their outer ends. That Morin should have designed an original form of dynamometer automatically recording the product of two quantities, which has been the model of several similar machines, which have been employed in very important official tests of competing machines, bears circumstantial testimony to his ability as an engineer.

Dynamometer by Easton and Anderson.

In a later form of this type of dynamometer made by Messrs. Easton and Anderson, of Erith,* curved springs were employed instead of straight ones, the curvatures being placed in opposite directions so that the effect of centrifugal force on the springs was minimised. The boss of the wheel, which was displaced by the deformation of the springs, was furnished

* Proceedings of the Institution of Mechanical Engineers, 1876; see *ibid.*, p. 199, W. E. Rich.

with a coarse-pitched double-thread screw which was in contact with a little cross-head, which passed through a slot in the shaft. The cross-head was free to move along the axis of the shaft in line with it. A rod attached to this cross-head projected from the shaft, and was connected to an integrating apparatus, of the disc and roller kind, described under the heading "Integrators."

The Transmission Dynamometer of William Froude.

To understand fully how belt dynamometers act several points must be observed. The belt, in order that it may adhere to the pulleys driven by it, must possess sufficient initial tension to prevent slipping. If no friction existed in the machine and it was in motion, this tension would be exactly the same, on the leading and trailing sides of the belt. If the machine opposes motion from both friction and imposed load, then the belt will be in greater tension on the leading side than on the trailing one; the difference of these tensions on the two sides of the belt enable motion to continue against the imposed load; so if this difference of tensions be known, the power transmitted can be at once determined, in the same manner as if the motion of the pulley were due to a force equal to this difference of tensions acting at its circumference.

"Hence the power consumed, or the units of work expended during any instant, will be the product of the difference in pounds between the tensions of the leading and trailing sides of the belt at that instant multiplied by the space in feet travelled by the belt in that instant."

The recording apparatus used in connection with this machine gives the sum of all such elements of work, and therefore the whole work done, during any certain period of time. The figures (73 and 74) show the machine in side and end elevation. The power is supplied by the pulley A and consumed by the pulley B, which drives the machine to be tested. The direction of motion is shown by the arrows. The belt conveying the power passes over the pulleys CD, the axles of which are carried on the opposite ends of the beam E at equal distances from the point about which it is free to vibrate. The upper and lower portions of the belt are kept

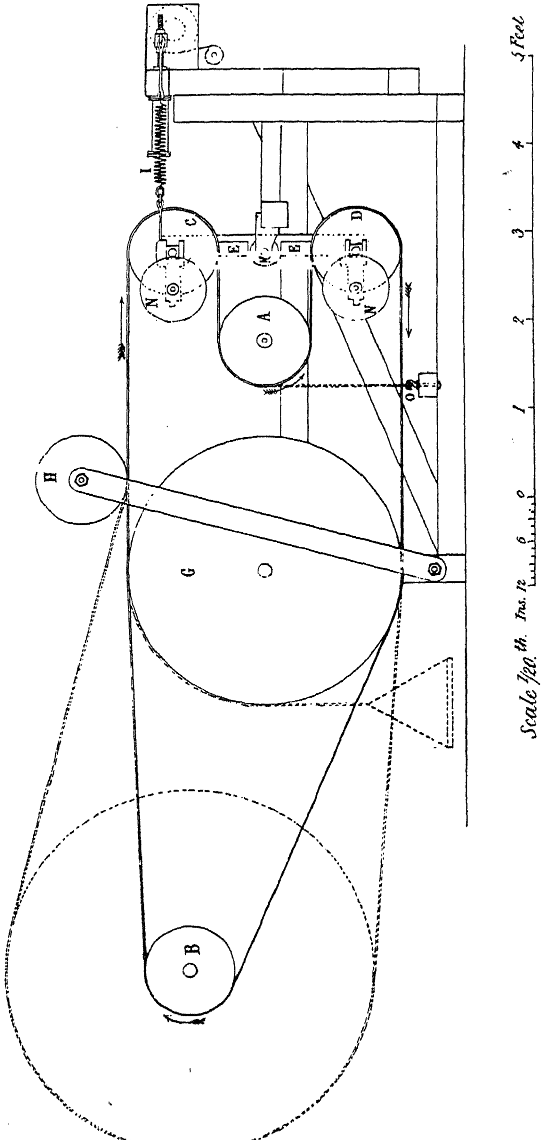
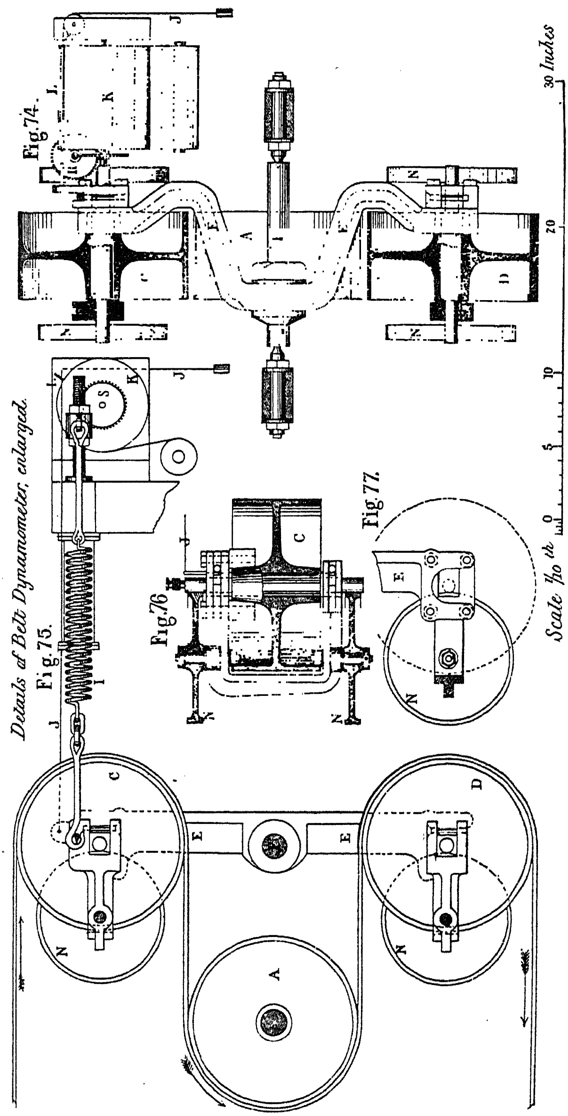


FIG. 73.

parallel, even when the beam is displaced through several degrees from the vertical, by means of the guide wheel G. The leading side of the belt passes round the pulley C on the



beam, and the trailing side round the pulley D; thus the pulley C is pressed horizontally by the tight side of the belt. The pulleys C and D are carried on axles which bear against antifriction rollers NN (Figs. 73—77).

"If the tensions of the two parts of the belt were equal, then the beam would remain in equilibrium, but since they are not equal, the spring balance I receives the resulting pressure of their inequality, and indicates its amount by its extension. The pressure is an exact measure of the difference of tension before mentioned. Now these tensions, which vary with the force at any instant absorbed by the machine under test, are continuously recorded on a band of paper travelling at a rate proportional to the belt speed; so that the area of the curve recorded, on integration, gives the whole work done in any set time. It must be noticed that the total tension on each of the pulleys carried by the beam is double that of the respective portions of the belt, since the tension of the belt acts on each side of the pulley; thus the spring balance indicates twice the difference of the tensions of the leading and trailing sides of the belt. Due allowance is made for this in the formation of the scale of force on the record traced on the cylinder. The speed of the recording cylinder is proportional to the belt speed, and represents it on a reduced scale.

"A pencil, deriving its motion from the beam E (Fig. 75), marks the extension of the spring balance on the moving cylinder. Since in this type of machine the spring balance has a tendency to oscillate on either side of its mean position, this motion is checked by the introduction of an oil dash-pot, that is, a cylinder having a loosely-fitting piston attached to the spring balance; the ends of the oil cylinder are connected by means of a tube, through which the oil can flow when conveniently checked by a stop-cock. The recording cylinder is provided with a continuous sheet of paper, which is uncoiled from a lower cylinder; and as it revolves the pencil traces on the paper a line, or rather a curvilinear area, in which each increment in length represents the corresponding space travelled by the belt, while the height of the point measured from the datum line traced by the pencil, when the spring balance is at zero, represents the stress upon the spring balance while the belt travelled through that space. The aggregate area included in any length of the diagram thus produced represents, therefore, exactly the units of work performed in that time; this is easily measured in the same manner as an indicator diagram, and can then be converted into units of work performed, when the scale by which the conversion is to be effected is determined."

In these tests the driving band is replaced by a continuous cord (mentioned in the introduction) running over grooved pulleys, these grooved pulleys corresponding to the belt pulleys of the transmission dynamometer described. In order that the pulleys might be relieved of friction, they were carried on a bent

bracket (Figs. 76 and 77) furnished with a friction roller N on each side. And to avoid any oblique deflection the arms of the swing balance were cranked (Fig. 74). This produced a corrective couple, and the axes of the pulleys were kept truly

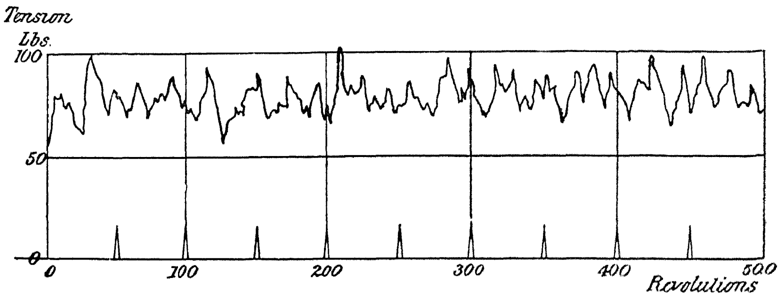


FIG. 78.

parallel to the axis of the beam. The leading side of the belt (Fig. 73) runs on the upper pulley C, and opposite, and on a level with it, the indicating apparatus K is placed. The belt used in this machine consisted of a double thickness of strong

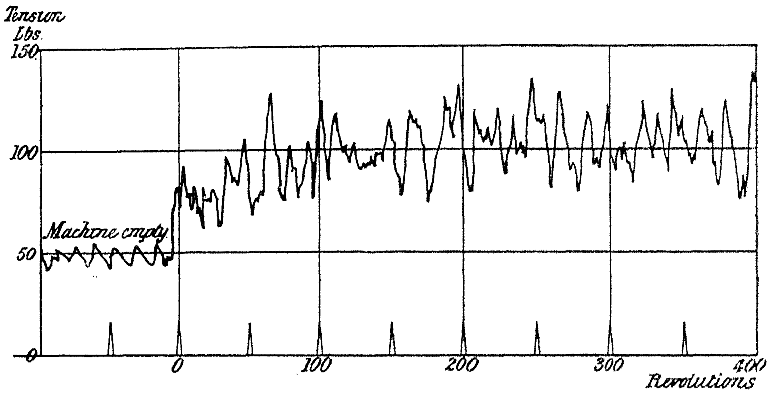


FIG. 79.

webbing sewed together and saturated with boiled oil. Such a belt proved itself to be remarkably supple, smooth, tenacious, and adhesive. Diagrams taken from trials are shown in the figures (78—80), which are portions of the actual traces made by the indicator pencil. The horizontal lines indicate tensions

of 50, 100, and 150 lb.; while the vertical lines are marked in intervals of 100 revolutions of the pulley on the beam. In Fig. 78 the diagram of power expended in thrashing barley is shown, the total mean tension being 85 lb.; Fig. 79 is another, for thrashing wheat, total mean tension 110 lb.; Fig. 80 is a diagram of chaff-cutting.

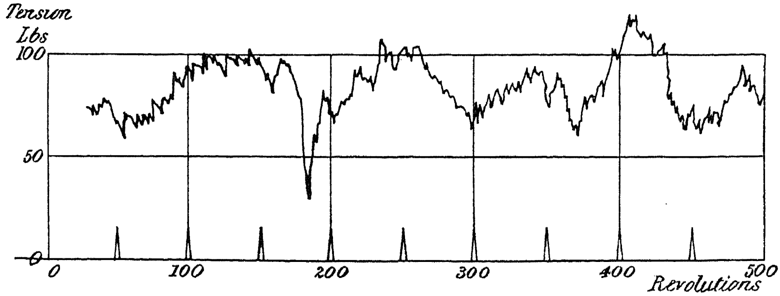


FIG. 80.

The following summary gives the relative value of work done in thrashing in each case:—

	Wheat.	Barley.
Number of sheaves thrashed .	200	200
Mean tension of belt (pounds) .	110	85
Total number of revolutions .	7,525	7,050
Total length of belt passed (feet).	23,636	22,148
Total units of power consumed (foot-pounds)	2,600,290	1,882,580
Units of power consumed to thrash one sheaf (foot-pounds)	13,001	9,413
Total time occupied in thrashing 200 sheaves (minutes) . .	11·40	10·68
Horse power for driving thrashing machine in work . . .	6·914	5·343

Horse power $= \frac{\text{Total foot-pounds.}}{\text{Total time} \times 33{,}000}$. Or foot-pounds per minute divided by Watts's constant of 33,000.

This description of the dynamometer of William Froude is practically an abstract of a paper in the Proceedings of the Institution of Mechanical Engineers, July 28, 1858, and I

have to thank that institution for permission to reproduce the figures.

The recording cylinder K is driven from the pulley C by means of two successive worm wheels R and S (Figs. 74, 75), and the speed is thus reduced to $\frac{1}{2500}$ of that of the pulley. The first wheel R is furnished with a cam, which moves another pencil close to the indicating pencil; this when unmoved by the cam traces a straight line along the paper, but at each revolution of the wheel the cam makes a narrow mark on this line, the length between the marks equalling 157 feet of belt.

Calibration of the diagrams.—The working belt is removed, and another belt is fixed to the point O in the frame. It passes over the power pulley, then round the pulley C, and over the guide pulley G. To the loose end of this belt weights are suspended. For determining any scale a weight suspended from the end of the belt gives the same stress on the spring as an effective driving force of the same amount. If now the paper is moved a little by hand, the position of the pencil for the load is marked on it. The reading of a diagram is greatly facilitated by causing eight pencils to rule lines along the diagram at distances from the base line showing different loads. It is also found convenient to rule lines of different colours which denote the different loads. Such coloured lines are now used in dynamometer diagrams produced in the tests of ship models.

I have given a full description of the dynamometer of William Froude in the first place because it is very clear and definite, and secondly because the method employed for obtaining a record of the measurement of the power transmitted is practically the same as that now used in the ship-model testing department of the Admiralty, and also naval departments of other nations who have followed the example of the British naval authorities.

TRANSMISSION DYNAMOMETER by the author, *Philosophical Magazine,* Vol. XV., p. 87; and "Dynamo Electric Machinery," by Professor S. P. Thompson, 1884, p. 383.

A steel shaft, tubular at each end (Fig. 81), and opened out

into a wide slot between the pulleys and one of the bearings, carries two pulleys, one keyed to the shaft and provided with two bevel wheels, the axes of which are in the plane of rotation

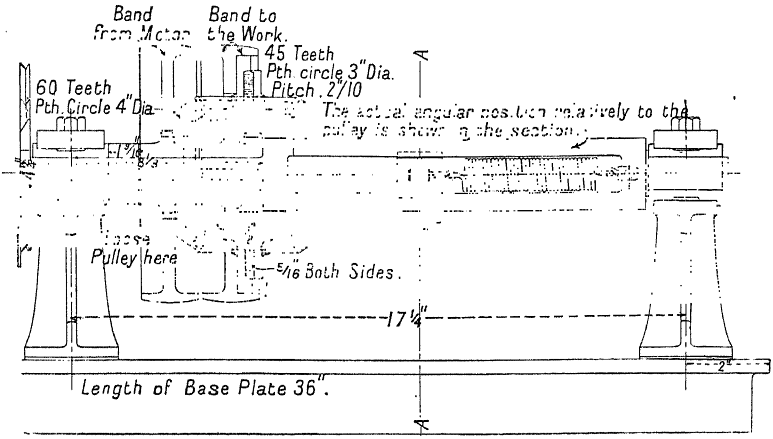


FIG. 81.

Section at AA.

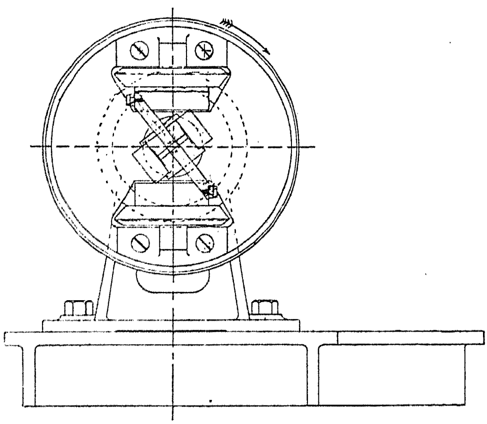


FIG. 82.

of the pulley; to the other a concentric bevel wheel is fixed, which engages with the two former ones. If the loose pulley is displaced with respect to the fixed one, the two bevel wheels are rotated, and by their rotation through connecting bands

attached to cylinders which form a part of each they extend a spiral spring the axis of which lies in the axis of the shaft. A rod attached to a cross-head which moves in the slot before

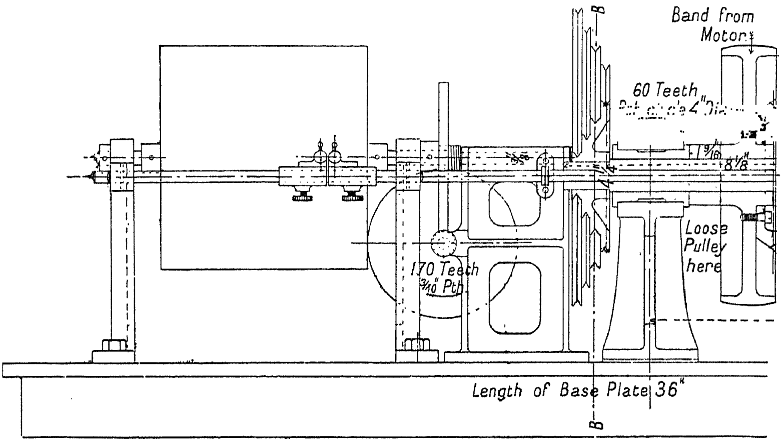


FIG. 83.

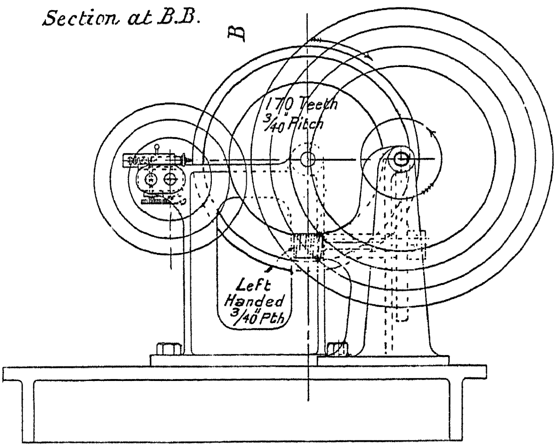


FIG. 84.

mentioned (Fig. 82), which is attached to the spiral spring and moves with it as it is extended, passes through the tubular end of the shaft, and to it is connected an integrating apparatus (described in the chapter on " Integrators ") which records the

power transmitted by the machine when driven by an engine and driving a machine to be tested for consumption of power. The same rod moves a pencil over a recording cylinder (Figs. 83 and 84) similar to that used in the dynamometer of W. Froude. The spiral spring was placed where shown in order to minimise the action of centrifugal force on it. In my later machines of this type, the cross-head is connected to the bevel wheels in such a manner that the flexible connection does not introduce any deformation due to centrifugal force which might interfere with the equilibrium of the system. Sometimes when the machine is driven by a turbine or motor at a uniform speed the force acting is shown immediately on a dial. The dynamometer was calibrated by the same method as that employed by W. Froude already described.

The author also employed another method of calibration, described in "Work-Measuring Machines" (Messrs. E. and F. N. Spon, 1884). It is as follows :—

"Let a prime-mover (a water-wheel appears to be most steady) drive the transmission ergometer, and let the ergometer drive a pulley on a shaft embraced by a Prony or other suitable friction ergometer, and let the work done against friction be calculated. This should agree with the results of the transmission machine. If it does we may conclude that it has been correctly calibrated. The advantage of this method is that the transmission machine is tested while running in its usual condition."

I have called this process of calibrating Dynamic Weighing by Taring.

Dynamic Weighing by Taring.

The method of weighing by taring, due to Borda, employed when the ordinary balance is used to determine the weight of any object is to place the object in one pan of the balance and then counterpoise it with any convenient material, such as shot and sand, placed in the other pan. The object is then removed and replaced with known weights which exactly equilibrate the counterpoise; then the sum of the known weights equals that of the object weighed. The advantage of the method is that error due to lack of equality in length of the arms of the balance is not introduced.

As an extension of this method, which may be shortly stated as making both A and B equal to C, and consequently equal to one another, I devised the following method of comparing the work done in driving any machine, such as a dynamo or screw propeller, with a definitely known amount of work done, under exactly the same circumstances, by the same prime-mover. An example of the application of the method may be considered. We wish to compare the power absorbed by a given machine such as a propeller with the power absorbed by a rope brake. An electric motor is attached to the propeller and the exact readings taken of the watts consumed during the run. Then the propeller is detached and replaced by a rope brake dynamometer. The former electrical readings are re-established and the brake horse power determined from the rope brake dynamometer. Then this brake horse power equals that absorbed by the propeller.

This method has shown itself by no means difficult to carry out; and when the electrical instruments are close reading, and dead beat, there is no reason to suspect it of error. The current is best read by reading the potential difference at the extremities of a known resistance in the supply circuit. The supply should be taken from accumulators so that the E.M.F. may be constant. It is nearly impossible to obtain a really constant E.M.F. from an ordinary town supply of electricity. The method suggested itself to me while testing a transmission dynamometer against a brake dynamometer. To make the method generally useful in the mechanical laboratory it was found necessary in some cases to alter the speed of the driving shaft. It is not a difficult matter to regulate the apparatus so that the speed of rotation may be the same in each case.

For reading the speed of the shaft either the optical methods of Lord Rayleigh * or Mr. Bosanquet are suitable. In the apparatus of Mr. Bosanquet a disc perforated with radial slots rotates on the shaft, and a tuning-fork, driven electrically, vibrates at right angles to the radial slots, close to the disc. When the combined motion is viewed by looking through the slots, and the number of slots passing per second equals the number of vibrations of the fork, a fixed wave line is visible, but if these numbers are not equal, the wave line shifts to the right

* Proc. Royal. Soc., 1881, p. 111.

or left, showing that the speed has either increased or decreased. This method is exceedingly accurate, and, though perhaps rather too physical in its nature to commend itself to the engineer, would well repay the experimenter for trouble expended on it. In the case of ordinary speed indicators we have to take them on trust, as correctly calibrated, and assume that their accuracy has been preserved, after daily use. In the optical method we have to rely on the time-keeping qualities of the tuning-fork. Experience extending over many years has shown that a well-made standard fork, such as those of Koenig, maintain their accuracy of vibration for very long periods, if preserved from any rust and injury. By employing a counter such as that of Harding and a well-rated stop-watch, the total revolutions during any set time may be found, but this method does not show any variations on either side of the mean speed ; the optical method, however, does show these variations in a very marked manner.

Transmission Dynamometer by Ayrton and Perry.*

In this dynamometer the two halves of a flange are connected by means of spiral springs, and when the shaft to which the half-flanges are fixed transmits power they are extended. The extension of the springs is shown by means of a bright bead attached to the end of an arm which is moved towards the axis. The background over which the bead moves is dead black. When the shaft rotates the bead describes a luminous circle, the radius depending on the extension of the springs. A scale having a sliding pointer enables its position to be read. The reading on the scale multiplied by the number of rotations per minute of the shaft gives at once the horse power actually passing through the coupling. The same principle is also embodied in a transmission dynamometer which can be moved, so as to test any fixed machine. In this apparatus the reading is made in the same manner as in the former one described, but the shaft is provided with three pulleys, two of which belong to the dynamometer proper, while the third is an idle pulley, on which the driving belt runs when the machine is not in action.

* "Applied Mechanics," by John Perry, F.R.S., Cassell & Co., 1897.

The Transmission Dynamometer of F. von Hefner Alteneck. *

In this apparatus a continuous driving band connects the engine with the machine under trial. About midway between the two pulleys a balanced frame carries two guide pulleys, both of which are in contact with the outside of the driving band. The distance between them is such that the band is constricted by passing between them. When the angle made by the following side of the band on either side of the guide pulley equals that made by the leading side on either side of its guide pulley, then a certain force is required to keep the frame and guide pulleys in a symmetrical position. This force was measured by a spring which was extended by means of a screw, until an index reached its normal position. The force thus found, divided by the sum of the sines of the two angles made with the centre line by the two parts of the driving band on each side of the same guide pulley, equalled the difference between the tensions of the leading and the trailing sides of the band. In another form of this dynamometer, in order to simplify the reading of the angles involved, the band was led over seven pulleys carried on a frame capable of displacement. To check vibration a small dash-pot was connected to the frame. No account is given of the friction which would arise from employing so many pulleys. One would imagine that it would out-balance the small inaccuracy due to determining the angle between the sides of the band and the central line. This machine did good service when used to find the efficiencies of dynamo-electric machines.

The Dynamometer of M. Matter (used by MM. Dolfus Mieg, of Mulhouse).

The interesting feature of this machine is the introduction of a power diagram, on which curves are traced which are the loci of points for which the product of effort multiplied by speed are constant. This is evidently the rectangular hyperbola. The power diagram is generated and used thus. The

* "Seprat-Abdruck aus den Bayerischen Industrie und Gewerbeblatt," 1883, heft I.

abscissæ are proportional to the velocity in metres or feet per second, and the ordinates to the force in kilograms or pounds. The loci of all points for which the product of these factors equals a constant lie on a curve of equal powers. Several such curves are carefully drawn on a surface so actuated by the dynamometer that displacement along the line of abscissæ is proportional to speed (this is effected by a speed indicator of the Buss type); at the same time the surface is moved at right angles to this direction by the machine, so that the ordinates are proportional to the force at any instant. The position of a stationary point in front of the diagram will be situated on, or near, a curve which at once shows the power transmitted by the dynamometer. For example :—Let equal distances along the line of abscissæ denote 1, 2, 3, ... metres per second, and the ordinates 10, 20, 30, ... kilograms, then, where a line, for instance, through 7·5 metres per second, cuts a line through 10 kilograms, we find a point on the curve such that their product equals 75 kilogram-metres, or one *force de cheval*, or French horse-power.

The Dynamometer of King.

In a dynamometer by King, which is quite unique, a spring of horse-shoe shape is placed actually in the driving belt connecting its ends; as this travelled with the belt it registered the tension by means of a pawl and ratchet wheel, the reading during a given time being that of alternate tensions. The original account of this apparatus is brief.

[The Dynamometer of C. V. Boys.]

[A transmission dynamometer described by C. V. Boys at a lecture at the Royal Institution * depends on the fact that if the driving belt is elastic, as for instance is the case with belts made of a close helix of steel wire, the spires are somewhat more open in the tight side than they are in the loose side. As the belt does not accumulate at one pulley, the linear velocity of the tight side must be greater than that of the loose side to such an extent that the same number of spires pass any point in a given time on the two sides. The tight side passes

* Proceedings of the Royal Institution, 1883, page 241.

on to the driving pulley, which accordingly moves at the speed of the tight side, while the loose side leads on to the driven pulley, which in the same way travels at the speed of the loose side. The driving pulley therefore turns somewhat more quickly than the driven pulley if they are of the same diameter, the difference being proportional to the driving couple. In order to record the difference in speed a crossed band from a pulley on the driving shaft was made to turn an equal pulley riding loose on the driven shaft at its own speed in the opposite direction. A differential gear connecting the two pulleys on the driven shaft then moved with a speed proportional to the rate at which work was being transmitted, for this speed is the product of the speed and the difference of speed or torque. Thus it automatically integrates the work transmitted, and time records of this give the average power in the intervals. This is not suited in the form described for measuring more than small power, but the same author has shown how by an epicyclic connection any known small fraction of the torque may be transferred from one shaft to the other by an elastic band, the rest being transferred by a nearly inelastic belt, such as the usual leather belt, or absolutely.]

The Dynamometer of M. Bourry.

In this machine the angular displacement of two pulleys by means of bell-crank levers compresses springs. The motion of compression acts on a disc which slides on the axle and actuates the integrating apparatus. The machine is furnished with a lever connected with the disc mentioned, designed to regulate the speed of the engine driving it ; so that it became a dynamometric governor.

The Dynamometer of M. Megy.

This machine, which was made by the firm of Sautter Lemonner, of Paris, consists of a horizontal axis carried on two bearings, beyond one of which it projected. To the projecting end a pulley was keyed. On the inner side of this bearing a loose pulley was placed ; a boss, bearing against the nave of the loose pulley and fixed to the shaft, carried two flat steel springs similar to those by Morin ; these were in contact

with the loose pulley near its face. An elongation of this boss which carried the springs was screwed with a quick pitched thread, which engaged with a nut which was rotated by means of two studs projecting from the loose wheel. When the loose wheel was displaced with respect to the fixed one the springs were deflected and the nut rotated, which caused it to traverse the shaft lengthwise. The motion was transmitted to the recording integrator of the disc and roller type. This machine did excellent service in the early days of testing dynamo-electric machinery.

The Dynamometer of Ruddick.

In this machine the parts which are displaced are fixed directly on the shaft. In several ways its action is similar to that of the Ayrton and Perry flange dynamometer. The displacement of a pulley with respect to a flange compresses springs, and the displacement is magnified by means of a lever carrying a pencil at its end, which marks a disc carried on the shaft. The pencil approaches the centre of the disc as the force transmitted increases. The disc is rotated by means of a ratchet mechanism moved at each rotation of the pulley.

The Dynamometer of Valet.

In the dynamometer of M. Valet the displacement of a pulley with respect to a shaft is shown by a recording apparatus, the springs in this machine being of the flat type. The recording apparatus rides on the shaft, and is prevented from rotating by a projection.

The Dynamometer of Neer.

In this machine the two halves of a flange coupling are connected to one another by means of four rollers placed at equal distances on the face of one flange, with their axes at right angles to it. Over these four link chains pass, one end of each being fixed to the other flange, and the other end of each chain being attached to a sleeve capable of movement along the shaft, and in so doing compressing eight spiral springs. The motion of the sleeve is very closely proportional

to the couple existing at any moment between the flanges. This and also the revolutions are shown on two dials.

The Dynamometer of M. Latchinoff.

Two pulleys are connected by means of spiral springs, and the displacement is read by utilising the phenomenon of the persistence of vision. One pulley is marked on the inner side of the face with divisions and the other with a single mark. The position of this mark on the curved scale is viewed through a slot in the face of the pulley, so that an image of its position is seen for an instant at each revolution. Although this dynamometer is not well known, it embodies a very interesting physical principle, which is bearing good fruit at the present time in connection with at least three forms of Torsion meters.

The Dynamometer of Tatham.

This machine in certain respects resembles the belt dynamometer of W. Froude. It consists of six pulleys, two of which are carried at equal distances from the fulcrum of a lever or frame free to move about a knife edge. Two are carried on bearings in the same vertical line, below those on the lever; each of these is on a shaft furnished with a pulley, one of which is driven by the prime-mover, while the other drives the machine under test. A continuous band passes round the four former pulleys, and the difference of tension in the band deflects the lever, to the end of which a steelyard type of balance is attached. The fulcrum of the balance can be adjusted so that it may be kept horizontal under the load to which it may be subjected. In the machine of W. Froude it will be remembered that careful provision was made for keeping the sides of the band parallel, so that changes in the angular position of the beam would not affect the effective working of the machine. At the same time it afforded a means of obtaining an automatic record of the power employed. In another dynamometer by Tatham, in place of the two pulleys carried on the single beam, two beams or levers are employed, and by each a pulley is carried. Each of these beams or frames is supported at their external ends on knife edges and planes, and their inner ends are connected by links to a weighing beam placed above them,

so that the difference of tension on the two pulleys can be found. In this machine the leading and trailing sides of the belt are parallel. It will be noticed that this condition is practically obtained by carrying the two upper pulleys on *two* beams which are free to move on pivots; at the same time provision is made for giving to the band sufficient initial tension for driving purposes. The band used in this machine was of leather, the joints being carefully and smoothly made. The flesh side was next to the driving pulleys and beam pulleys, and the hair side next to the upper central pulley. If a belt is fairly long, and narrow, one of its surfaces can always be used in contact with three pulleys. All that has to be done is to give the belt a half-turn on either side of the intermediate pulley, then the same side of the belt will touch all the pulleys. [A belt with a half twist in it is specially suitable for driving between pulleys on shafts which make a large angle with one another. Such a form of belt is well known as having one surface and one edge only.]

The Dynamometer of M. Farcot.

The principle underlying the mode of action of this machine is practically that of both Froude and Tatham. The sides of the belt are kept approximately parallel and two tension pulleys carried on separate levers or frames placed under the driving pulley instead of above it, as in the dynamometer of Tatham.

The Dynamometer of Parsons.

In a dynamometer of Parsons* the same idea is found. Vertically below a grooved driving pulley the driven pulley is placed. A continuous rope passes over the driving wheel, then round a pulley in a block from which a weight hangs, then round the driven pulley, and then round a pulley in a block back to the driven pulley. The difference of the tensions of the two sides of the rope is found by taking the difference of the values of the suspended weights required to establish equilibrium when the machine is running. This dynamometer was used by its inventor in connection with experiments on screw propellers.

* Proceedings of the Institution of Mechanical Engineers, 1877, Hon. R. C. Parsons.

The Marine Dynamometer of Saurin.

In this machine, which is amongst the earliest employed for measuring the output of a marine engine, the springs, which are deflected by the imposed couple, are of peculiar shape. Mr. Gisbert Kapp, in his excellent articles on "Dynamometers" in *The Electrician* of January 19th, 1884, makes the following remarks on this dynamometer, which he attributes to M. Saurin (in *La Lumière Électrique* the name is spelt "Taurines"):—

"If a straight spring of uniform section be held rigidly at one end, and the deflecting force applied at its free end in a direction perpendicular to its length (similar to the load on a cantilever), then a minimum of force will produce a maximum deflection. If both ends are supported and the load applied in the middle, the deflection will be only one-quarter of what it was before. Or, for the same weight of steel employed and with an equal deflection, the force to be measured can be quadrupled. This arrangement will, therefore, be better than M. Morin's original plan of straight radial springs, if large powers are to be transmitted. But M. Saurin goes a step farther. He employs springs slightly curved, and applies the power in such direction as to pull the ends apart and thus straighten the springs. With this arrangement a minimum weight of steel can register a maximum pull at a moderate deflection, as indicated by the more or less complete straightening of the bent springs. The propeller shaft is divided and connected by means of an elastic coupling, consisting of two cross-bars and two curved springs. One cross-bar is on the engine side of the shaft, the other on the propeller side. Studs project from the ends of these cross-bars, which engage with the ends of the curved springs. The springs are placed with their convex sides outwards from the shaft. The effect of a separation of the ends of the cross-bars is to pull the springs out and make them straight. The resistance to straightening is obviously very great, and this form of spring apparently provides the condition of minimum weight for maximum transmission of energy. The displacement of the springs makes an automatic record of force while a fixed pencil draws a datum line from which to reckon the value of ordinates."

The Dynamometer of Professor Dalby.

A pulley drives a shaft through a spiral spring. The displacement of the pulley with respect to the shaft is indicated

by means of a very interesting mechanism. Two equal sprocket wheels are attached, the one to the spring pulley, the other to the shaft. An endless band of steel passing over them forms two loops, which remain at the same distance apart when the system is rotating, but if angular displacement takes place between the two sprocket wheels their distance is changed, and this change is proportional to the torque transmitted by the shaft. In order to measure this, two guide pulleys are placed in the loops, guided by a geometric slide. One of these pulleys carries the scale and the other an index. A reading on the index is proportional to the torque. Should oscillations occur they may be damped by the introduction of a dash-pot, or, which is better, practically prevented by employing a relatively stiff spring.

[The Dynamometer of Amsler.]

[The transmission dynamometers of Dr. Alfred Amsler were shown at the meeting of the Institution of Mechanical Engineers held in Zürich on July 25, 1911, and an illustrated account of them is given in the Proceedings of that Society for that date, pp. 603—616. I am indebted to the courtesy of the Institution for permission to reproduce here the figures representing the construction of the larger of the two, as also one of the torsion dynamometers by the same constructor in a subsequent chapter. Fig. 85 is sufficient to show the operation of this machine. Two pulleys D and B are mounted on the same shaft C, B being keyed and D running loose upon the shaft. Power is communicated to the pulley D by means of a belt, and a second belt transfers the power from the pulley B to the driven machine. The connection between the two pulleys is effected by means of projections J from the pulley D pressing upon the ends of two pistons working in cylinders carried by the pulley B. These pistons are not packed, but are made a smooth fit, and leakage and friction are both immaterial. The pressure in the oil behind the pistons is taken by means of the curved pipes G to the hollow axle and thence through an axial fixed tube, which enters the axle without leakage by means of a stuffing-box to the casing N. To this is connected a pressure-gauge O, and also the piston of an indicator similar to those

used for indicating steam engines. The paper is fed continuously past the pin by worm gearing in proportion to the speed of the shaft. At high speeds the centrifugal force of the oil in the pipes G may interfere, but it is possible to balance this for one position only of the pistons. For this reason Dr. Amsler prefers to use this type at moderate speeds. The

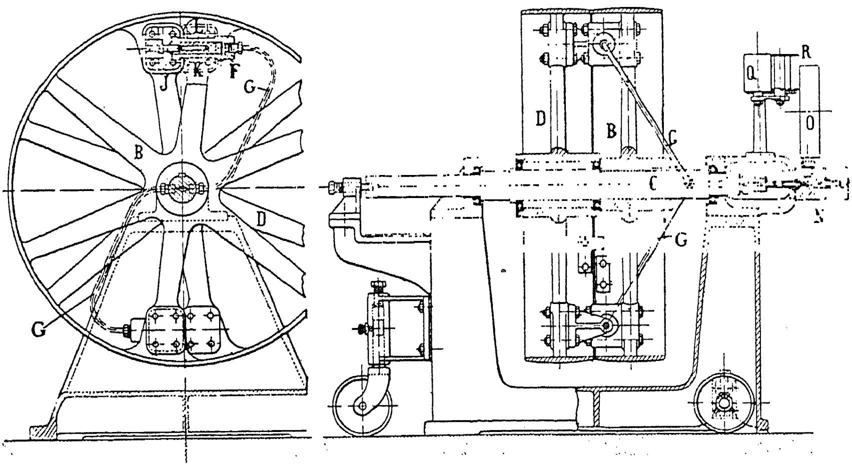


FIG. 85.

two dynamometers were designed for torques of 50 and 150 metre-kilograms (4,340 and 13,020 inch-pounds) respectively.]

[MOORE'S DIRECT-READING ELECTRICAL DYNAMOMETER.]

[An interesting and original method of employing electrical means not only to measure the angle through which a spring connecting the two shafts of a transmission dynamometer is twisted, and hence the driving torque, but also automatically to multiply this by the speed, so that the power being transmitted may be read at any moment upon a voltmeter, is described by Mr. C. R. Moore in an article on a "Direct-Reading Electrical Dynamometer" in the *Electrical World* (New York) for 1912, p. 449.

Each shaft carries an alternator with a two-pole field magnet,

and the exciting current sent through the two in series is necessarily the same for each. In all other respects the two machines are made identical, and they are so designed as to give very accurately a simple harmonic wave form to the alternating electro-motive force which they induce. The perfection of this result is shown by an oscillograph record. The two machines are so connected and adjusted that they are in exactly opposite phases when there is no torque and consequently no twist in the spring. When, however, one shaft is transmitting power to the other, the exact opposition of phase no longer obtains. In consequence of this the circuit, which consists of the two armatures in series and a voltmeter, is no longer dead, but an outstanding voltage is available to act on the voltmeter; and this voltage is proportional to the torque multiplied by the speed, so the readings of the voltmeter at once give the power being transmitted at any moment, and the two independent readings of speed and torque need not be made. Further, by the use of a switch, one of the connections can be reversed, then the voltmeter which indicates the vector sum of the two separate voltages or either voltage may be read so as to ascertain the speed. The proof of the proposition is simple. Where two equal harmonically varying quantities having the same period are compounded, the resultant is a harmonically varying quantity of the same period and of an amplitude which is zero when the components are in exactly opposite phase; which is their arithmetical sum when they are in the same phase, and which, when there is a small departure from identity or opposition, is in the first case very slightly changed, while in the second it is proportional, with considerable exactness to the departure from exact opposition.

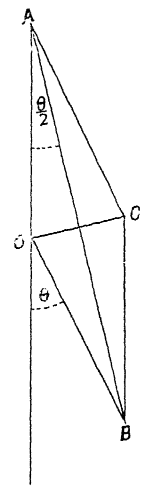


FIG. 86.

This may be shown by reference to Fig. 86. Taking the two equal vectors OA, OB, nearly in opposite directions, but differing from this by the angle θ, their component is OC, and this is equal to $2\,\mathrm{OA} \times \sin \theta/2$.

As, then, θ changes from 0 to a moderate angle, $\sin \theta/2$

changes so as to be very nearly proportional to θ, as the following figures indicate:

θ	Sin $\theta/2$.	Arc $\theta/2$.	Per cent. error.
5°	·043619	·043635	·0367
10°	·087156	·087270	·1307
15°	·130526	·130905	·2900
20°	·173648	·174540	·5110
25°	·216440	·218175	·7950

Thus, for angles up to 20 degrees the departure from strict proportionality is only $\frac{1}{2}$ per cent., and it is less than $\frac{4}{5}$ per cent. at 25 degrees. OC then is the amplitude of the harmonic wave, which is indicated by the voltmeter, and this is proportional not only to the angle θ, but also to the absolute magnitude of OA or OB. As these are proportional to the speed the indication of the voltmeter is proportional to the torque multiplied by the speed or to the power being transmitted. By the use of the switch one phase may be reversed, then AB instead of OC is shown on the voltmeter; or one component alone may be read, the other being cut out, and this at once gives the speed. Provision is made for adjusting the phase of one of the alternators so as to obtain exact opposition of phase, or this may be adjusted so that the voltmeter reads zero when the dynamometer is running, so as to eliminate the small losses therein. It will of course be clear that the electrical load due to the alternators is infinitesimal, for the only output is that needed to actuate a voltmeter; the load practically undiminished is transmitted to a recipient machine. A diagram is given showing the extreme accuracy of the straight-line law both for speed and for power when tested against a Prony brake. The machine is set up in the electrical laboratories of the Purdue University.]

[F. W. LANCHESTER'S WORM DRIVE DYNAMOMETER.]

[F. W. Lanchester's worm drive dynamometer is in a sense a transmission dynamometer, but not in the sense in which that term is generally used. The transmission dynamometers

described so far have been appliances to measure the power transmitted by some intermediate connection from one machine to another, but this machine of Lanchester's is designed to measure directly the efficiency of a worm drive, *i.e.*, the ratio of the power transmitted to that received. The valuable paper describing this unique machine will be found in the Proceedings of the Institution of Automobile Engineers for the session 1912—13, Vol. VII., p. 238, and I have to thank that Institution for permission to reproduce Fig. 88.

When power is transmitted from one shaft to another by means of worm gearing, the torques in the two shafts, which

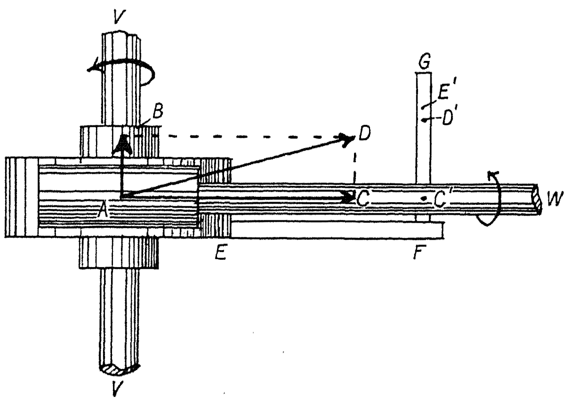


FIG. 87.

would be in the inverse ratio of their speeds of rotation if there were no friction, must, as action and reaction are equal and opposite, be impressed in the opposite sense on the worm gear casing. Taking a usual case of the worm drive of the back axle of a motor car with a reduction in speed of, say, 4 to 1, the torque on the worm shaft would be one quarter that of the back axle if there were no friction. Actually, as there is some friction, it is a little more than one quarter. It is the object of the worm drive dynamometer to determine the proportion which the ideal torque $\frac{1}{4}$ bears to the real torque $1/r$, then $\frac{1}{4} \div 1/r$ or $r/4$ is the efficiency of the gear. If there were no friction in the gear the casing would simultaneously be subject to torques of 4 : 1, about two axes parallel to the axis of the worm wheel and of the worm respectively. Fig. 87 is

an ideal representation of the worm gear casing seen from below with the worm shaft W and the back axle VV. Supposing the two shafts to be turning in the directions indicated by the arrows round them, the worm being the driver, then the casing must be subject simultaneously to two torques about these axes, the smaller one of which is represented on the diagram by the arrow AB acting at the end of an arm of a given length standing up from the centre of the worm shaft, while the larger one is represented by the longer arrow AC acting on an arm of the same length as the former one standing up from the centre of the back axle. These two are equivalent to a single force AD acting on an arm of the same length as the other two standing up from a point about which the casing may be supposed to be supported. If to the casing a rigid bar EF is fastened and this is made to carry another bar FG parallel to the back axle, then the two torques upon the casing can be neutralised by the application of a single force to the bar at the point D′ in the line of AD and directly away from the paper —*i.e.*, if the casing is so supported as to have any freedom of rotation about each of the two axes or about the point A, which in the plan is their crossing point. With a 4 to 1 reduction the distance C′D′ would be exactly one quarter of C′A, if there were no friction in the gear. As there is some friction, the distance CE′ at which a single force will neutralise both torques must be a little more than one quarter of C′A. In the actual machine, of which a photograph is shown in Fig. 88, the worm casing is supported on ball or roller bearings about the axes of the two shafts and the two distances C′A, C′D′ are 24 and 6 in. respectively. The heavy weight hung from the knife edge D applies a counter-acting torque about the two axes simultaneously, and the value of the smaller torque may be adjusted by means of the screw with head E and counter F attached. Ring bolts are secured to the base so as to prevent the heavy weight from tilting the frame through more than a small angle about either axis. Dash-pots are fitted to damp out vibrations about either axis. The shaft on the left-hand side of the photograph is the worm shaft; the shaft of the worm wheel which goes out at the back is not visible in the photograph. Flexible couplings are fitted to both shafts to allow the necessary

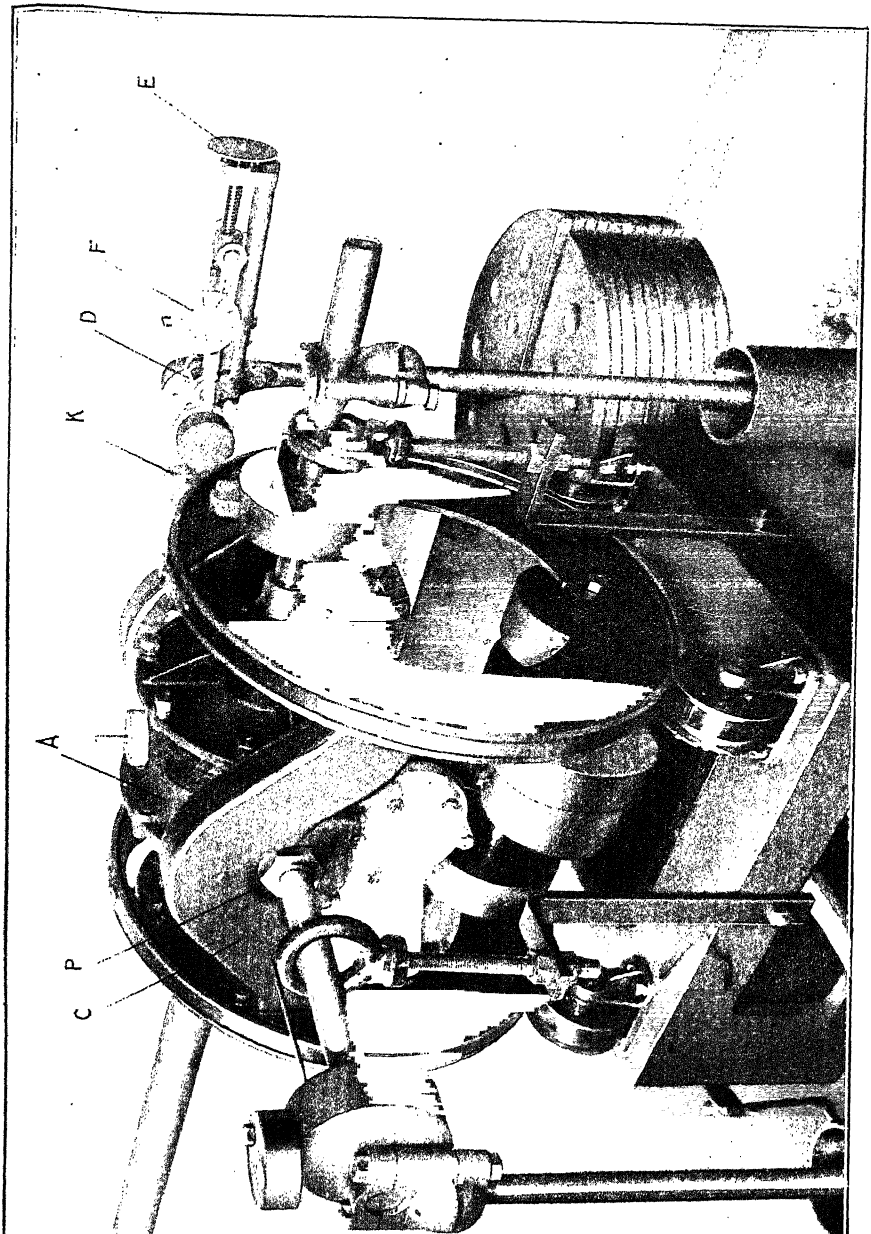


FIG. 88.

freedom in the casing. In using the dynamometer power is applied to the worm shaft by a four-cylinder motor-car engine, and the worm wheel shaft is made to transmit its power back to the worm shaft through the intervention of a slipping belt running just too fast for the worm shaft. In this way the engine is only called upon to supply the power lost in the whole of the mechanism. The power passed through the gear depends partly on the speed and partly on the tightness and consequent friction of the slipping belt, and these are capable of independent adjustment.

In making a test the complete swivel framing is first carefully balanced with the shafts at rest. The worm is then set into rotation and the weight hung at C′, and there adjusted in amount until the frame is balanced about the worm wheel axis and the steady rod is floating in its eye-bolt. The torque in inch-pounds in the worm wheel shaft is then found by multiplying the number of pounds in this weight by 24. The knife edge is then screwed along to some point E′ beyond D′ of Fig. 87, until the secured steady rod floats in its eye-bolt. When this is the case the torques on the two shafts are in the ratio of AC′ to C′E′, whereas if there were no friction it would be in the relation of AC′ to C′D′, and C′D′/C′E′ is the efficiency.

Supposing the efficiency to be 95 per cent., then C′D′/C′E′ = ·95, or since C′D′ = 6 inches, C′E′ = 6·322 inches
with efficiency 96 per cent. C′E′ = 6·248 ,,
with efficiency 97 per cent. C′E′ = 6·186 ,,

As the position of the knife edge at which the frame floats can be obtained with an accuracy of about $\frac{1}{100}$ inch, it is possible to determine efficiencies with an accuracy of $\frac{1}{5}$ per cent.

The results obtained with this apparatus have proved to be of the highest value. They have shown that the Lanchester worm gear is unsurpassed in efficiency and that unexpectedly great pressure may be taken by the worm without loss of efficiency, and that this may reach the high value of 96·8 per cent., representing a loss of only 3·2 per cent. The efficiency is affected slightly, as might be expected, both by the speed and by the torque transmitted; but for further information on the results the reader is referred to the original paper. It may, however, be well to mention that the friction with different oils and at different temperatures was found to depend upon

something besides the viscosity, and pure mineral oil was not so good as animal or vegetable oil; in fact, the frictional losses were in some cases nearly double as great with mineral oil as with animal or vegetable oils.

It is clear that the Lanchester machine could be modified so as to test the efficiency of bevel gearing.]

[Draw-Bar Dynamometers.]

[The author has left no account of draw-bar dynamometry, such as is practised in the dynamometer coach of an experimental train. Strictly this is "dynamometry," and it comes more nearly under the heading of "transmission dynamometers" than any other. He acknowledges however on page 15 the permission given to him to reproduce a figure of the dynamometer car of the Great Western Railway. I do not know whether the omission by the author to proceed with this was intentional, as taking him too far from his main subject, or not. I do not care, therefore, to do more than mention the existence of this class of testing and to refer in particular to two recent publications. The first is by Prof. W. E. Dalby, in the Proceedings of the Institution of Mechanical Engineers, 1912—14, in which the observations made in a trial run are treated very completely by graphical representation. The second will be found in a series of articles in the *Engineer* for the year 1913 by Mr. C. R. King on "The Dynamometry of Locomotives, with special reference to the Use of Super-heated Steam."]

CHAPTER XI

TORSION POWER-MEASURING MACHINES OR TORSION METERS

RECENT forms of power-measuring machines of the Torsion type, designed to measure the shaft horse-power delivered by marine engines, have been the outcome of the difficulty experienced in indicating a steam turbine in a satisfactory manner. The necessity of employing such machines is well and clearly shown in the first paragraph of a paper read on March 21, 1907, by Mr. Archibald Denny before the Institution of Naval Architects. He writes thus :—

"When the suitability of the turbine method of propulsion for commercial work was proved by the success of the *King Edward*, built by my firm in 1901, it became apparent to us that it would be highly desirable to have a method of ascertaining the horse-power transmitted by the turbine shafts to the propellers.

"Until that problem was solved we could only work from the boiler to the propeller, and the efficiency of the turbine and the propeller must be lumped together. It is not possible to 'indicate' the turbine in the same way as is done for a piston engine, although I may say that a fair approximation can be got by ascertaining the fall of pressure through successive expansions by means of pressure gauges fixed to the turbine casing."

Before describing the torsion meters of different inventors we may briefly examine the principles underlying the construction of power-measuring machines of this type.

If by means of a shaft of elastic material (well within its

elastic limit) the energy of a prime-mover is imparted to a machine such as a rolling mill, or to the propeller of a steamship, and the following quantities are known—namely, the twisting moment of the elastic shaft T, in statical inch-pounds, the number of revolutions of the shaft per minute N, and the angular motion of the shaft $2\pi N$—then the horse-power transmitted is

$$\text{H.P.} = \frac{2\pi \text{NT}}{33{,}000 \times 12} = 0{\cdot}00001586 \text{ NT}.$$

The function of this type of machine is to show, by some means, the angle of torsion, which represents a given statical twisting moment, when the shaft is rotating under load. For effecting this, optical, mechanical, and electrical methods have been pressed into service. The angle of torsion θ for a given statical twisting moment T is first found when the shaft is at

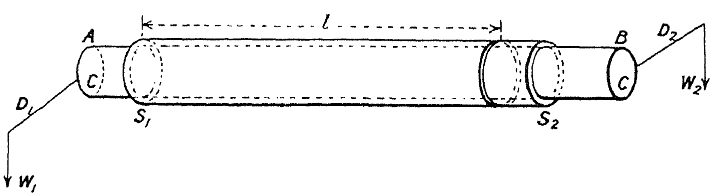


FIG. 89.

rest, and from this determination other independent values of θ and T are known, since $\theta \propto$ T.

One method whereby the calibration may be made is illustrated diagrammatically in Fig. 89, in which AB is the shaft, supported on centres C, C. The weights W_1, W_2 act on the shaft through the arms D_1, D_2. A tubular sleeve is fixed to the shaft at S_1, and another short sleeve is fixed to the shaft at S_2. In order that an exactly known length of shaft may be dealt with, the sleeves are fixed on the shaft by means of three equidistant pointed set screws which engage with the shaft on a line traced round it. In some of the earliest experiments on torsion this was the method employed for fixing the sleeves on a measured length of shaft. Recently annular edges have been employed to fix the sleeves, the sleeves being kept in position while they are clamped on by bolts and distance pieces (the method used

in the Hopkinson-Thring torsion meter). When these are removed, the two sleeves are left on the shaft in exact position at a known distance l apart. Known weights W_1, W_2 are caused to act on the shaft and the deflection noted where the edges of the sleeves meet, so that a known twisting moment is indicated by the angle of torsion due to it, which can be read. Now, from the law connecting the twisting moment with the angle of torsion, we know that when the angle for any other twisting moment is read the moment is at once known, providing always that the twisting moment is well within the elastic limit of the material of which the shaft is made. Since the relative motion between the adjacent ends of the sleeves is very small, the reading is usually effected by means of some magnifying device, either optical, mechanical, or electrical.

The angle of twist in the case of a torsional ergometer is best found by actual trial when the instrument is set up. But for the purpose of finding θ very approximately, as an aid in designing, the following method of calculation may be employed. The angle of twist θ can be found from the dimensions of the shaft transmitting a given H.P. at a given number of revolutions per minute, thus :—

Let—

the diameter of the shaft $= d$ inches,
the twisting moment $=$ T in statical inch-pounds,
the length of shaft under torsion $=$ L inches,
the revolutions per minute $=$ N,
transverse elasticity * $=$ G in millions of pounds per square inch,

then the angular velocity $= 2\pi N$
and the horse-power transmitted is

$$\text{H.P.} = \frac{2\pi NT}{12 \times 33{,}000} \quad . \quad . \quad . \quad (1)$$

and

$$T = \frac{33{,}000 \times 12 \times \text{H.P.}}{2\pi N} \quad . \quad . \quad . \quad (2)$$

now $T = fZ_t$, where $f =$ the greatest shearing stress
,, $Z_t =$ torsional modulus of the shaft derived from the dimensions of the shaft ;

* This term G has been called "the modulus of rigidity." By Prof. Unwin it is called "the coefficient of transverse elasticity" ("Machine Design," Part I., p. 50, 1909).

for a cylindrical shaft diameter d,

$$Z_t = \frac{\pi}{16} d^3 = 0{\cdot}196\, d^3,$$

for a tubular shaft

$$Z_t = \frac{\pi}{16} \frac{d_1^4 - d_2^4}{d_1} = 0{\cdot}196 \frac{d_1^4 - d_2^4}{d_1}$$

where d_1 and d_2 denote the outside and inside diameters.

For a cylindrical shaft

$$\theta = \frac{2TL}{GZ_t d} = \frac{2TL}{G\frac{\pi}{16}d^3 d} = \frac{32TL}{G\pi d^4} \quad . \quad . \quad . \quad (3)$$

Should the reader wish to find G, without very great accuracy in the reading of the length and angle of deflection being aimed at, the experiment can be made without any special apparatus on a small shaft mounted in a back-geared lathe, the fixed end of the shaft being held in a three-jaw chuck and the gear locked to prevent rotation. Near the ends of the shaft metal bosses carrying pointers of thin sheet steel are clamped. The bosses should be short and the section such that only an annular edge embraces the shaft. The edges of the steel pointers are in the plane of the annular edges of the bosses. This construction provides an easy way for placing the plane of the pointers at a given distance from fixed points on the rod. Beyond the boss, at the back-centre end of the lathe, a pulley is fixed on the shaft embraced by a flexible belt, such as webbing, from which weights are suspended to produce the desired torsion. The edges of the steel pointers move against vertical scales. When the shaft is unloaded the pointers should be horizontal, and the points in which they cut the vertical scales taken as the zero of each. The weights are put on in front of the lathe and the difference of the readings taken. The pointers should reach about 30 inches from the axis of the shaft towards the back of the lathe bed. For finding the tangent of the angle of deflection, and from it the circular measure of the angle, it will be found convenient to employ metric scales, the distance from the axis of the shaft to the vertical scales being also measured in millimeters. In an experiment in which the shaft and apparatus were mounted as described the length of the torsion arm, namely, the radius of the pulley = 5·27 inches; the load on its end = 2·2046 pounds; the torsional moment = 5·27 × 2·2046

inch-pounds; the distance from the fixed end to the boss between the pointers $L = 25\cdot59$ inches; the radius of the small shaft $r = d/2 = 0\cdot1165$ inch; $\tan\theta = 0\cdot09327$; θ in circular measure $= 0\cdot09308$; $\pi = 3\cdot14159$.

The connection between the quantities involved is embodied in the equation

$$G = \frac{32TL}{\theta\pi d^4};$$

putting the values found into this equation we find that

$$G = 10\cdot512 \times 10^6 \text{ pound inch square.}$$

From equations (2) and (3)

$$H.P = \frac{2\pi NT}{12 \times 33{,}000}$$

$$T = \frac{\theta G\pi d^4}{32L}.$$

If the torsional angle is read in degrees and written θ°, it must be converted to circular measure by the divisor 57·3, so that

$$H.P = \frac{\theta^\circ N d^4}{L} \times \frac{2\pi^2 G}{12 \times 33{,}000 \times 57\cdot3 \times 32}$$

$$= \frac{\theta^\circ N d^4}{L} \times 0\cdot3262.$$

So that in each dynamometric experiment on the same shaft only θ° and N have to be recorded.

CHAPTER XII

TORSION POWER-MEASURING MACHINES OF DIFFERENT INVENTORS

IN this form of power-measuring machine, as I have shown, the torsional angle of a shaft must be known while it is rotating and energy is transmitted by it. Hirn * appears to have been the first to employ this direct method of measuring power. The torsional angle in his machine is shown by means of a pointer deflected by means of differential cog-wheel gear, the number of cog-wheels being eight.

At each end of a known length of shaft a cog-wheel is fixed; these are in gear with cog-wheels keyed on two small counter-shafts placed parallel to the main shaft, rotated in opposite directions by means of an intermediate cog-wheel. Bevel cog-wheels of a differential gear are keyed to these shafts, and

* "Les Pandymomètres," par G. A. Hirn: Paris, Gautier Villars, 1876.

a pointer is fixed to the frame which carries the intermediate bevel wheel, the axis of which is at right angles to the axes of the two former ones.

The differential gear, consisting of two bevel cog-wheels, one loose the other fixed on the same axle, each being in gear with a third bevel wheel, the axle of which is at right angles with the former axle, appears to be the invention of H. Holdsworth, who patented the device (1826) in connection with the winding machinery used in the manufacture of cotton yarn. Since that date the method has been constantly employed in many different kinds of machines, such as the driving mechanism of telescopes, the differential governor of Siemens, and the driving gear of traction engines, tricycles and motor cars. In the case of vehicles it enables the two driven wheels to rotate with different angular velocities while they are both being driven in the same direction. The mechanism is commonly known as "Jack in the box." But to return to the torsion machine of Hirn. When the main shaft revolves without torsion, the rotation of the two bevel wheels of the differential gear is equal, but if torsion is set up in the main shaft, one cog-wheel advances on the other, and the frame of the intermediate bevel wheel is deflected and with it the pointer, which shows the angle of torsion.

The end of the pointer is hinged to a light lever, which actuates the recording wheel of an integrator of the type employed by Morin, so that by this means the power transmitted during long periods of time may be estimated. This very interesting original paper should be read in order to appreciate the genius of Hirn.

In 1893 I devised a differential gear for showing the torsion of a shaft transmitting work, in which only four wheels and a flexible joint were employed. In Hirn's apparatus eight wheels were required.

The machine was exhibited at the Soirée of the Royal Society (exhibit 22, p. 11 Descriptive Catalogue, Conversazione, Royal Society, May 2, 1894).

RQ is the shaft (Fig. 90), LM its bearings, ABCD are gun-metal wheels of equal diameter made with involute teeth. The rod K below the shaft is carried in the bearings BN.

The wheel D is carried on the arm E, which is free to rotate about the axis of the shaft on a concentric sleeve; the wheel D is driven by means of the double flexible joint FG, not a Hooke's joint.

If the system be rotated when no work is being transmitted, then the pointer P remains at the zero of a divided dial, but when the shaft is subjected to torsion the point P is deflected through an angle proportional to the torsional angle in a plane perpendicular to the plane of the paper. Also, since the axis of the wheel B is fixed when the system is rotating, the pointer P indicates the angle of torsion. The arm E can be connected to an integrator by means of which the whole work done during any time may be estimated. This form of differential gear appears to require the least number of moving parts. The

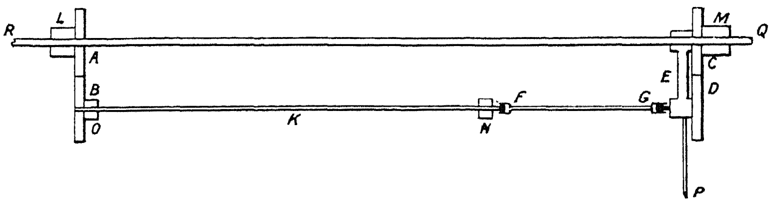


FIG. 90.

differential method of indicating the angle of torsion of a propeller shaft has been used by the author, and the results obtained show that the apparatus is practically dead-beat. The gear may also be used to indicate electrically when a certain limit of torsional angle is exceeded.

Two optical methods of reading the torsion of a wire or shaft while rotating were shown at the Royal Society, May 2, 1894.

The first depends on the phenomenon of the retention of an image by the organs of vision for a fraction of a second. The second method depends on the reversal of the motion of the image of a rotating object by means of a combination of mirrors revolving at half the angular velocity of the object and in the same direction.

The first method is illustrated by an application of the optical principle to an instrument used to measure the work done in rotating a copper cylinder in a magnetic field. The

arrangement of the apparatus is shown only diagrammatically in Fig. 91.

A copper cylinder C is attached to a torsion rod or wire SS; this wire is attached to the upper end of a tube which runs in the bearings BB; the tube carries a cylindrical scale DD at its lower end, divided at its edge into degrees. The torsion rod is furnished with a double pointer PP, which is turned up at its ends so as to pass closely over the divisions of the scale on the cylinder; the torsion wire at its lower end passes through an agate collar A fixed to the tube; by means of the pulley W the whole system is rotated about a vertical axis.

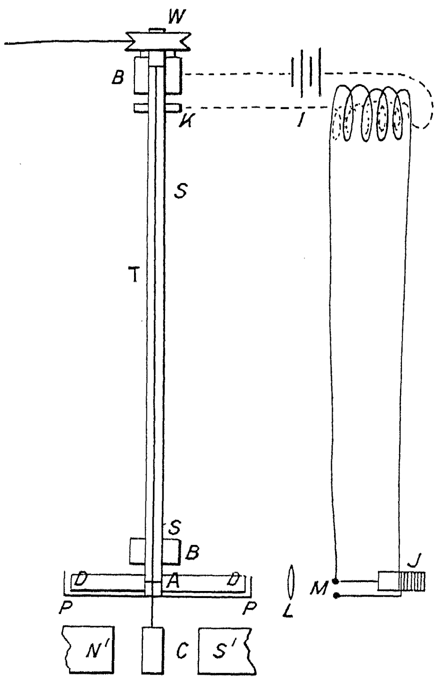


FIG. 91.

The tube T carries an electric break K, and the primary coil of the induction coil I is twice closed and opened at each revolution. The secondary wires go to a Leyden jar J and a spark at M illuminates the pointers and scale; the light due to the spark is concentrated by the lens L. Thus at whatever speed the system is rotating the spark is always made at the *right instant*, and thus the position of the pointer can be easily read. A small vacuum tube may be used at M, but the illumination is not so good as that due to the spark. This method of illumination enables the experimentalist to use the torsion balance while rotating at any required velocity.

The name "optical rotostat" has been given by me to a combination of mirrors so arranged and moved that a rotating object viewed by reflection from their surfaces appears to

be at rest. It is described in *Engineering* thus :—" The rotostat is an instrument for optically bringing to rest the image of revolving objects, such, for example, as the spokes of a revolving wheel." The instrument, used in conjunction with a torsional work-measuring machine, was exhibited at the same date (May 2, 1884) at the Royal Society. Since that time it has been applied to a variety of uses. I find that prior to my experiment an optical arrangement for viewing mixed colours had been employed by Lord Rayleigh. In this case the coloured discs were fixed, and their *mixing* was produced by the rotation of an inverting prism. The description of the method is most interesting, and I give it at full length.*

" In conclusion I will describe an apparatus by which it is possible to observe these colour-matches without rotating the disks. . . . The idea, which I carried out . . . was to spin an *image* of the disks instead of the disks themselves. An inverting prism was mounted in a tube which could be made to rotate. The axis of rotation is adjusted so as to point accurately to the centres of the disks mounted as usual. An eye applied to the prism sees the disks undisplaced as a whole, but inverted by reflection. As the tube rotates, the image of the disks rotates also, and with double angular velocity. When the speed is sufficient, the colours lying on any circle concentric with the disks are blended exactly as if the discs themselves revolved."

The author was unaware of the existence of this method when he used the combination of mirrors for reading the dial of the ergometer when rotating ; although in each experiment practically a reversing prism is used, yet they differ in this respect—namely, that in the experiment cited the instrument is used to mix that which is seen through it, whereas in the author's application of the reversing mirrors or prism the apparatus is used for optically bringing to rest a divided scale rotating before it, by rotating it at half the speed of the object viewed and in the same direction as the object is rotating. If the reflector is a prism this is mounted in a tube which can rotate about its own axis. A ray of light passing along this axis is reflected as shown by the broken line DE from the face BC of the prism ABC. (Fig. 92.)

* British Association Report, September 2, 1881, page 46.

I find that the rotostat is capable of another useful application, namely, to show when two engines are running at the same speed. The reflecting prism is rotated by one engine, while a disc with a line ruled on it as a diameter is rotated at half the speed of the former by another engine. If the two engines are running exactly at the same speed, the line does not appear to move, but if one gains or loses on the other, the line is rotated at a rate proportional to the difference of their speeds.

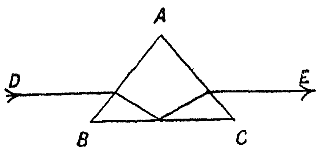


FIG. 92.

[I used this device in order to bring the rotating film of the rainbow cup * to apparent rest, and showed how, with slight errors of centering, curious trochoidal disturbances are set up. The reflecting prism used as described by the author is not really exactly equivalent to a plane mirror, for it reflects a conical bundle of rays with its axis parallel to the reflecting surface (Fig. 93), bringing those rays that come from below up over the reflecting surface, by which they are totally reflected, and then allowing them to continue their path below this plane again. This, of course, is impossible with a plane mirror. As it is important that the reflecting

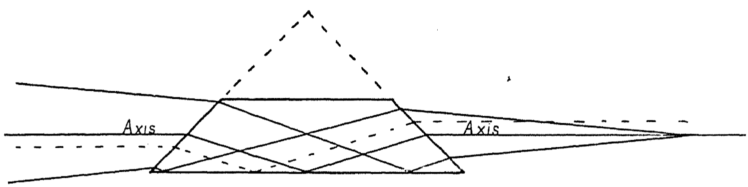


FIG. 93.

prism should be accurately placed in its tubular support, not only with the long edges of the reflecting plane parallel to the axis, but with this plane the right distance from the axis, I give the position calculated for the ordinary refractive index of glass, which is 1·5. With a right-angle prism of such glass the reflecting surface should be distant from the axis of rotation by an amount equal to ·1125 of the length of the hypotheneuse face. All that part of the prism near the right angle, which is

* Proceedings of the Royal Society, Vol. LXXXVII., 1912, page 349.

more than double this distance from the reflecting face, shown dotted in the figure, is outside the limit of rays entering the prism parallel to the axis which can be reflected from the larger face and is useless. If the reflecting face is not the correct distance from the axis of rotation, a central ray parallel to this acquires a lateral deviation, as shown by the dotted ray path in the figure. If the prism is so set that its reflecting surface is not exactly parallel with the axis, an angular deviation results, while if the axis about which the prism turns is not directed towards the centre of the rotating object which it is sought to bring to apparent rest, the object will appear to turn about an eccentric point at twice the speed of rotation of the prism, whereas the other errors give apparent motions at the same speed as the prism and the combination of errors leads to trochoidal curves, which, while full of interest, destroy the illusion.]

During certain experimental work in which it was necessary that the energy transmitted by a rotating shaft should be known, I devised in 1909 the following method of reading the angle of twist of the shaft, and found that it gave good results. It is easily applied and might be used for other similar purposes in the mechanical laboratory. Clear readings can be taken from a pointer moving over a circular dial which reflects light, while it rotates about a diameter. The optical principle involved is similar to that of the Thaumatrope of Dr. Paris, in which on one side of a card the head of a man was painted, and on the other side a hat. When the card was rotated by means of twisted strings attached to the opposite edges of the card, the head and hat appeared as one picture—the *rationale* of the experiment being that the picture of the head is retained by the organs of vision until the hat appears, the two separate impressions thus making one picture. In my apparatus the reflecting dial, made of mirror glass, is fixed so that its plane is parallel with the axis of the shaft or of the spiral spring, the angle of twist of which is to be measured. The mirror is perforated in the centre, and through the perforation the pivot which carries the pointer passes. The pointer is deflected (by means of connecting links attached to the shaft, and also to a sleeve fixed to the shaft) through an angle proportional to that of the twist of the shaft.

A parallel beam of light is projected on to the mirror dial by means of two plano-convex lenses, as used in a projection lantern, the source of light being an arc lamp slightly shielded by ground glass. The image of the pointer, moving over the divided scale, can either be viewed direct by one person by reflection, or, which is far more convenient, the image can be projected on to a screen, by means of two achromatic lenses, and then it may be viewed simultaneously by several observers. This latter method of viewing the pointer is preferable to the former one, which is rather fatiguing to the eye owing to the intermittent flashes. Even when the speed of the shaft is slow, about 300 revolutions per minute, and the flashes occur at intervals of one-fifth of a second, the image of the pointer is clear and well defined. I have applied this optical method of reading a moving dial to a torsion work-measuring machine placed between an electric motor and a dynamo feeding an arc lamp. The best way of reading the torsional angle of a shaft is undoubtedly by means of an automatic record or trace made on a paper-covered cylinder driven from the shaft by a scribing pen the motion of which is proportional to the force ordinate at any instant and hence to the angle of torsion. The speed of rotation of the cylinder is reduced by means of gearing, so that a record extending over a long period of time may be taken.

In some cases such an elaborate method of reading the torsional angle is not required, and the optical method I have described is sufficient for those cases in which only rather small powers are dealt with, as, for example, in aeroplane engines and motor-launch internal combustion engines.

Electrical Methods of Reading the Torsional Angle of a Shaft.

The February number of the *Philosophical Magazine*, 1898, contains the description of an electrical method devised by myself whereby the angle of torsion of an elastic shaft was determined while rotating. The method was applied to find the torsion of a long shaft used in driving a dynamo, and also the torsion of a solenoidal spring used as a flexible shaft to drive propellers of different forms under different conditions of

immersion. Two discs of insulating material were fixed near to the ends of the shaft. Each disc was furnished with a narrow contact-piece at its edge, connected to the shaft and two metal brushes (one of which was stationary and the other moveable) pressed on the discs as they revolved. An electric circuit was formed including the shaft, a line-wire, a battery, and a telephone. When the shaft was at rest and the brush was touching the contact-piece, on the disc at the driven end of the shaft, the brush at the driving end of the shaft was then adjusted by being moved on an arm which rotated about the axis of the shaft, so that on making or breaking the circuit a click was heard in the telephone ; this position of the arm and brush was marked as the zero from which the angle of torsion was reckoned. The shaft was then fixed at the driven end and subjected to a known statical torsional moment, and the brush rotated till a click was heard on making or breaking the circuit ; thus a dial indicating the torsion of the shaft was calibrated. Since the angle of torsion is proportional to the statical moment, within certain limits, the dial was easily calibrated when one or more points on it had been determined. When the power of an engine was transmitted through the shaft to a dynamo, the engine being at one end of the shaft and the dynamo at the other end, the arm carrying the brush was moved over the divided dial till the click was again heard in the telephone. The angle through which it was moved was thus the angle of torsion of the shaft. And the value of any torsional moment was found from the indicated angle of torsion. If the torsional moment in statical inch-pounds is T, the number of revolutions per minute N, and the horse-power transmitted H.P.,

$$\text{H.P.} = \frac{2\pi \text{NT}}{33{,}000 \times 12}.$$

Mr. Archibald Denny, not at the time of his experiments knowing of the existence of the paper I have mentioned, worked on almost exactly the same lines on workshop shafts, and also applied the method to the steamship *Queen Alexandra*, as described in a paper read before the Institution of Naval Architects, March 2, 1907.

The experiments of Mr. Denny are of so much value and interest that I have given a description from the paper cited,

which shows further developments tending to great accuracy of reading the torsional angle :—

"Some fifteen years ago we had made numerous experiments with factory shafting, endeavouring to ascertain the absolute torsion of a shaft while running, and it therefore immediately occurred to me that this was the proper direction in which to attack the problem. We had tried various methods, principally using pierced discs and beams of light, but with very partial success. We had not tried any method involving the use of electricity, and I therefore arranged for experiments to be made by this method on one of our factory shafts. The first trials were made by fixing discs on the shaft at a considerable distance apart, so as to get a reasonable amount of torque. The discs were of insulating material, and each had a contact point arranged at its periphery in such a manner that the point made momentary contact with a metal tongue or brush once in every revolution of the shaft. The contact points were connected to the shaft and the metal brushes to a battery and a telephone receiver. The method adopted was first to adjust the brushes, so that both made contact with the points simultaneously when the shaft was revolving but transmitting no power. When transmitting power the shaft was, of course, subject to a certain amount of torsion, and thus the brushes were put out of simultaneous contact. One of the brushes was then moved round its disc concentrically, until simultaneous contact was once more established. The amount of this shift gave a measure of the torque on the shaft, and to ascertain the correct amount of this shift the telephone receiver was placed to the ear, no sound being heard except when both brushes were in contact with the respective contact points, when a loud 'tick' was heard. The principle was thus of extreme simplicity, and the method of carrying it out seemed at first equally simple; indeed, I may say that this first rough apparatus, which was quite successful, only cost a few shillings to make. We then set about making more accurate and elaborate apparatus on the same lines to be fitted to the *Queen Alexandra*, which was nearly ready for trial, with an assured hope of getting satisfactory results.

"The factory shaft on which we made the original experiments ran about 120 revolutions per minute, but the revolutions of the *Queen Alexandra's* side shafts were over 700, and when we came to make experiments at this high speed we found the new apparatus was useless, as no certain sound could be got. We tried many forms of contacts, and after numerous experiments we did succeed in the *Queen Alexandra*, with revolutions about 750, in getting some fairly consistent results; but it was impossible to be quite certain of the exact point at which the

make and break in the circuit took place, and we were never quite sure of our results : still, we had made a great step in advance."

(Probably this difficulty of adjustment was due to the inherent property of a shaft, when driven by a reciprocating engine at high speed, of oscillating about its axis at a certain definite rate for a given rate of rotation. These oscillations are imposed on the torque of the shaft while rotating, and under certain conditions have a marked lag, with respect to the torsional moment at any instant, a point clearly shown by Dr. Föttinger, F.J.J.S.)

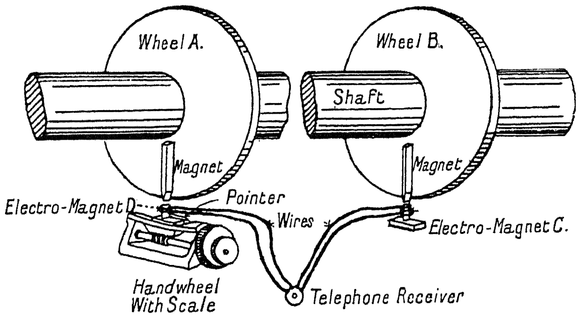


FIG. 94.

" Mr. Charles Johnson, a member of our staff, who assisted in working out this problem and was closely connected with it from the first, thoroughly appreciated the difficulties, and realised the desirability of getting away from the unreliable rubbing contact, and he ultimately succeeded in solving the problem in a most ingenious way.

" Fig. 94 shows his original solution. Two gun-metal wheels, A and B, were fastened to the shaft at a definite and known distance apart, the distance being as great as possible. On each wheel a permanent magnet, with a sharp chisel-shaped edge, was fixed radially at the periphery of the wheel and with the sharp edge parallel to the shaft. At one end a soft iron electromagnet C, wound with fine wire, similarly chisel-shaped, was fixed, so that the moving magnet passed directly over the electromagnet once in each revolution. At the other end a similar electromagnet D was mounted on a screwed sector, and wires from these electromagnets were led to a differentially-wound telephone receiver. If the shaft revolved without transmitting power, the permanent magnets passed these electromagnets simultaneously, and

currents of electricity generated in each coil passed through the telephone receiver, but, the currents being equal and opposite, no sound was heard. When the shaft transmitted power, the permanent magnets passed the electromagnets at different times, and hence a sound was heard in the receiver. By turning the hand-wheel shown in diagram, a new position of silence could be obtained, when it was evident that the two permanent magnets were again passing the electromagnets simultaneously, and the amount of torque could be ascertained from the reading of the sector screw."

A further development of this machine and apparatus is described under the heading, "The Denny-Johnson Torsion-Meter."

Most interesting and valuable researches have been made by Mr. Hermann Frahm on the torsional stresses developed in propeller shafts when running on ships. The work, which began in 1899, has led to the invention amongst other things, of the now well-known Frahm Speed-indicator, the action of which is based on resonance. A large number of minute white squares form the ends of thin steel reeds of different pitch ; these are clearly visible on a dark background. These steel reeds are attached to a shaft which is made to vibrate by means of an electromagnet, the current being supplied from a small alternate current dynamo of the induction type, driven by the engine the speed of which is to be known. The vibration is imparted to all the reeds, but those reeds only whose natural period of vibration synchronises with the period of the alternate current are thrown into vibration sufficient to be seen. This instrument may be placed at any convenient distance from the engine, and out of reach of other disturbances.

Mr. Frahm attacked the problem of discovering why propeller shafts were apparently exposed to stresses far in excess of what was generally supposed by marine engineers. It has been the case that propeller shafts have been fractured in a quiet sea, while such fractures did not appear to be due to defective material, nor did it appear likely that they could be due to couples, caused by steam pressure, since such forces seldom reached a dangerous value.

In November, 1899, Messrs. Blohm and Voss began an experimental investigation of the subject, which was conducted by Mr. Hermann Frahm. Since the distance between

the prime-mover and the screw is usually great, the propeller shaft could not be regarded as a rigid body: it is in fact a kind of spiral spring constantly changing in torsion as power is being transmitted by it when driven by any reciprocating engine. The matter would be different if the shaft were driven either by an electric motor or by a turbine. The torsion of the shaft would be then practically constant. But when the source of power changes, so also must the torsion of the shaft. But the changes in the torsions will not be proportional to the torsional stresses, as commonly assumed. It would be exceedingly difficult to predict what dimensions such changes would have. The torsional stresses in the shaft both as to magnitude and variation had to be found first. This was done by measuring in one revolution of the shaft, step by step, the torsions at the instant, in their absolute amounts. The torsional stresses could then be estimated. The next step was to obtain simultaneous readings of the changes of velocity of the whole system, both for the engine and the screw. After many preliminary experiments the following methods of working were adopted:—

Thin sheets of zinc foil were wrapped round the flanges, which were as far apart as possible. A scribing point of platinum, carried by a lever, pressed against each zinc foil, the pivot of the lever being supported on a nut on a screwed spindle placed parallel with the propeller shaft. An electric motor, the speed of which could be regulated, was placed close to the flange. The shaft of the motor carried two equal contact discs which made and broke the current in two independent circuits. These were led from the positive conductor to the contact-breakers and the platinum points through resistances. The zinc foil was coated with a black oxide; this was removed by the platinum point as long as the current acted. The method of making the experiment was as follows. When the high-pressure piston was at its highest point, the engine was stopped and the exact position of the platinum points marked, and this was the zero position, and in order to obtain this point exactly and as far as possible free from the friction of the bearing of the shaft one mark was made when the engine was stopped after moving in one direction and another made after moving the engine in the opposite direction; the

mean position between the two marks was taken as not far from the true zero.

The engine was run for some time before the experiments were commenced. The electric interrupter was then made to run at the correct speed and the electric circuits were closed. The levers were then, at a given signal, brought down on to the zinc foils and again removed at another signal; during this time spiral curves were drawn on the zinc foils, broken up at definite distances by the breaking of the current. The breaks gave data for plotting velocity curves for the two flanges, equal time spaces being marked by the beginnings of the breaks. In order to plot the velocity curve these distances, as marked on the foils, were set up as ordinates, the corresponding abscissæ denoting time. The curve plotted through the tops of these ordinates is the velocity curve.

In a second experiment the torsions, and hence the torsional stresses, were found thus. The zinc foils were placed in abutment, so that the marks for zero torsion and the top position of the high-pressure piston were on the same perpendicular. The relative angular displacements of the style marks, which corresponded to the same instant, marked the torsion of the shaft, which lay between the flanges. These torsions when plotted graphically led to a curve which showed the changes in the turning moments. The mean turning moments were calculated from the mean amplitude of this curve, and hence the power given to the propeller (the modulus of elasticity of the steel being known). Three shafts were made by three different firms—namely, F. Krupp, Bochumer Verein, and Gewerbschaft Wilkowitz—and tested at the Royal Mechanical Testing Station, Charlottenburg, and the mean value of the modulus of elasticity for thrust found by torsion tests. The modulus of the three shafts varied very slightly, and equalled 828,000 kilograms per square centimetre. Using this number the effective horse-power transmitted was calculated, and also the efficiency of the engines, that is, the ratio of the effective horse-power to the indicated horse-power.

Mr. Frahm states that, so far as he was aware, these were the first experiments in which the brake horse-power of engines of several thousand horse-power have been accurately found by using the shaft as a dynamometer.

THE TORSION METER OF DR. FÖTTINGER.*

In this torsion meter two sleeves or tubes embrace the shaft to which they are fixed at their remote ends (Fig. 95). The free ends of the sleeves where they nearly abut are furnished with discs which form parts of the sleeves. When the shaft is subject to twist, points on the edges of the discs initially opposite to one another are displaced through the angle of torsion for a given length of shaft.

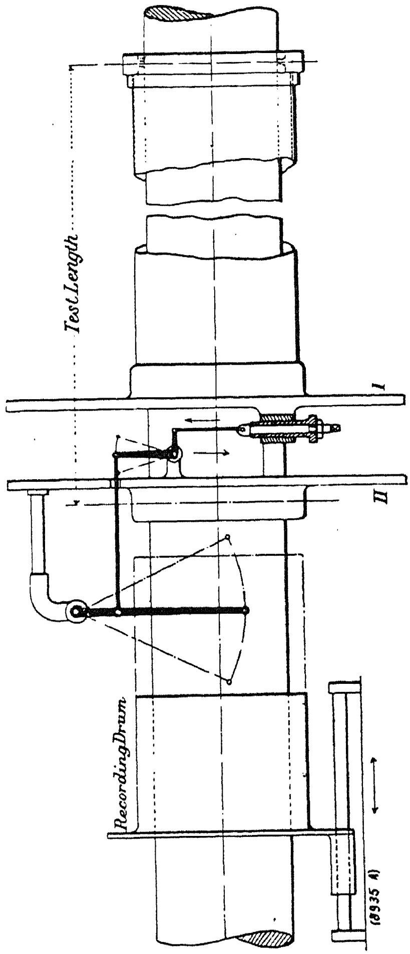


FIG. 95.

By means of levers linked to the two discs I, II, this angle of torsion is magnified about thirty times and recorded by means of a tracing point on a cylinder which rotates concentrically with the shaft but at a convenient rate more slowly than the shaft, the reduction of speed being effected by gearing. For example, when the ratio was 1 to 4, the length of one diagram gave the curves due to four revolutions of the shaft. The record of no torque is a circumferential line traced on the cylinder, or a straight line when the record is laid out on a plane ; it is the zero line from which the ordinates of torque are measured, and the area of the diagram is proportional

* Schiffbautechnische Gesellschaft, 1903.

to work done. This can be integrated, as already explained at p. 64. When the apparatus is used on a shaft driven by a reciprocating engine, the varying torque is shown as an undulating line—on one side of the zero line when the ship is going ahead and on the other side of the zero line when going astern. Some idea of the reading for a given diameter and length of shaft may be formed from the following details relating to the *Kaiser Wilhelm II*.

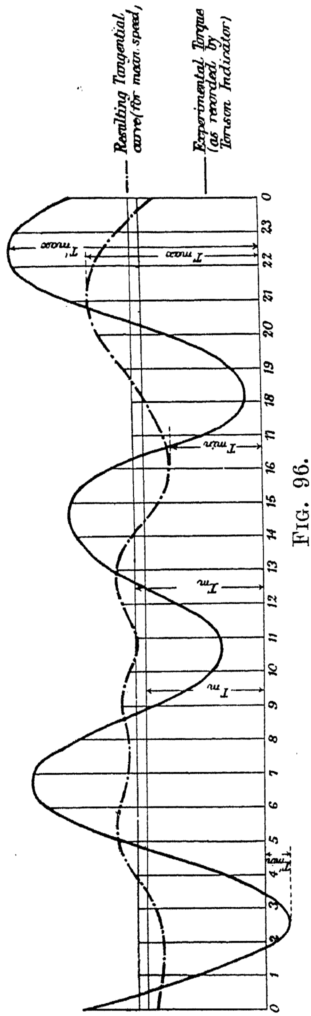


FIG. 96.

The diameter of the shaft was 604 millimetres = 23·78 inches;

the length of the shaft was 2,200 millimetres = 86·614 inches;

test radius on discs of the shaft was 550 millimetres = 21·653 inches;

revolutions per minute of the shaft was 80;

amplitude of curves obtained was 40 millimetres = 1·574 inches;

the magnification of the lever system was 27.

Two loose cylinders were employed driven by sun-and-planet gearing. When the engine employed to drive the propeller is either an electric motor or a turbine, then the mean effective torque is represented by a fairly straight line. As speed increases, so do troubles due to centrifugal forces, and these forces had to be contended with and carefully balanced: these important details appear to have been effectively worked out by the inventor. By means of this torsion meter and ordinary indicator diagrams sixteen in number, both the shaft horse-power and the indicated horse-power were found, and hence the efficiency of the engine for different values of

the power. In the case of the steamship mentioned, on her first voyage an efficiency of 95 per cent. was reached, and on a subsequent voyage an efficiency of 93 per cent.

In Fig. 96 is shown a curve recorded by the torsion indicator (continuous line), from which has been deduced the tangential curve for mean speed (broken line). There is apparently a curious difficulty in obtaining the true reading of torque diagrams, and comparing their ordinates with the tangential force, when the engines are running. If the diagram is marked O when, say, one of the cranks is highest and the engine is at rest, when the engine is working and the shaft is stressed, this point will be shifted, the shaft having travelled past its dead point by the amount of the angle of torsion at the instant when scribing point cuts the mark. Light can, I believe, be thrown on this rather difficult point by making the crank print a mark on the diagram, when it is at either its highest or lowest point, by electrical means. This would mark the shifted zero. The diagrams clearly show the existence of natural vibrations, developed at certain shaft speeds. When the torque diagram is integrated by means of a planimeter, the mean torque and the effective power can be calculated.

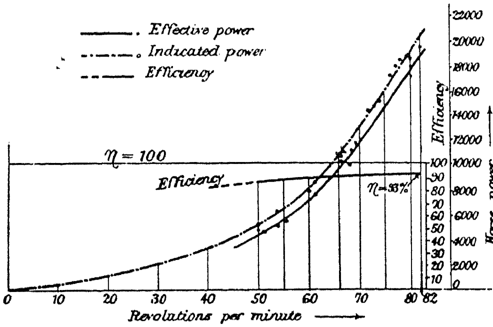


FIG. 97.

In Fig. 97 the efficiency curve is shown. It indicates a satisfactory result: down to one-fifth of the maximum horse-power the efficiency curve is nearly straight, and at 95 per cent. for a range between 20,000 and 10,000 h.p. Points on the efficiency curve are found thus. The effective power, as found by the torsion meter, is divided by the indicated power, deduced from the readings of all the steam indicators. The quotient gives the length of the efficiency ordinate.

Mr. Herman Föttinger's communication to the Schiffbautechnische Gesellschaft should be well studied by those who wish

to appreciate very exact and careful work, on the difficult problem of dealing with power transmitted by shafts which have a natural period of vibration at certain critical speeds.

The Denny-Johnson Torsion Meter.

In this torsion meter two sleeves or tubes are fixed on a propeller shaft, their ends being at a known distance apart; as in some other forms of torsion meters, one sleeve is long and the other short. Where they abut they do not touch, but are furnished with projecting arms. When the shaft is transmitting power, and subjected to torsion, the two arms move relatively to one another, the displacement being proportional to the torsion of the shaft, the length of which extends between the circumferences embraced by the two sleeves. On one arm of a sleeve the primary coil and core of a small transformer is fixed, and the secondary coil of the transformer is attached to the arm of the other sleeve, while a small air-gap separates the adjacent ends of the cores of the transformer. From this it will be seen that the air-gap length changes with the angle of torsion. The currents through the transformer are so led by means of slip-rings to brushes and return wires or earths (if the framework of a ship may so be called) that the primary may be excited by a small motor-driven alternator, while the secondary is connected to an alternate current voltmeter; so that the reading of the voltmeter is proportional to the angle of torsion and the voltmeter becomes the torque meter. The torque meter is calibrated in fractions of an inch of air-gap length and therefore of torsion, and the scales are divided in tenths, hundredths, and thousandths of an inch, the smallest division being easily subdivisible by eye; the error of observation is said to be practically *nil.*

The Torsion Meter of Prof. B. Hopkinson, F.R.S., and Mr. L. G. P. Thring.

The principle of this apparatus is a differential one, and depends on the observation of the twist of a shaft between two adjacent points on it by means of two beams of light projected from a fixed and a movable mirror on to a graduated scale. The beam of light projected by the fixed mirror determines the

zero point of the scale, and that projected by the movable mirror indicates the amount of torque of the shaft while rotating and driving the propeller of a ship. Although both mirrors rotate with the shaft, even at moderate speeds the

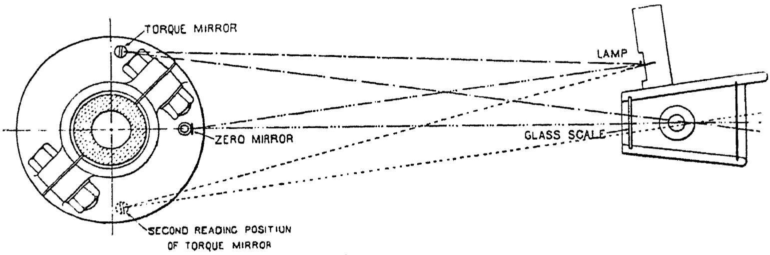


FIG. 98.

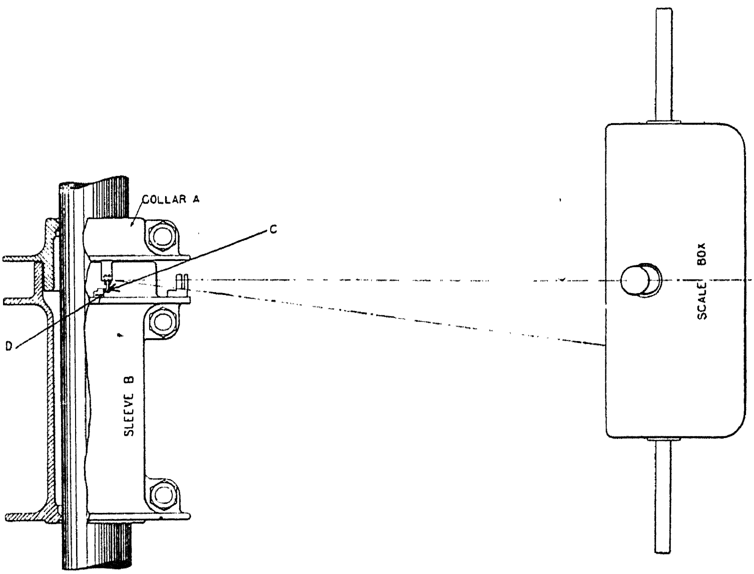


FIG. 99.

reflections appear as lines of light across the scale, which can be easily read. When this form of torsion meter is applied to a shaft driven by a turbine engine the reflected line of light is steady. Changes of torque would be due only to varying resistance experienced by the propeller through the change of

the position of the blades, in the case of the ship pitching. Even when reciprocating engines are employed, a good estimate can be made of the imposed torque, which, of course, varies during each revolution of the shaft. The construction and application of this torsion meter is shown in the two figures (98 and 99). A collar A, which is clamped to the shaft, is provided with a flange projecting at right angles to it and a tubular extension. A sleeve B, also having a similar flange and tubular extension, abuts against the collar. The collar and the sleeve are rigid, so that when the shaft is subject to a twist when transmitting power the flange of B moves relatively to the flange of A. This displacement is indicated by a mirror called the "torque mirror," actuated by a small lever C pressed by a spring on to a projection D. The axis about which the mirror rotates is at right angles to the axis of the shaft. The relative movement of the flanges rotates the torque mirror through a small angle, and deflects a beam of light over a divided scale, a reflection being received at each half-revolution of the shaft, since the mirror reflects from both of its surfaces. Fig. 98 shows the shaft in section and the disposition of the optical apparatus.

Prof. Hopkinson recommends a direct calibration of the shaft, with the instrument in the position in which it will be used before the shaft is put into the ship. This recommendation is excellent and sound, and more accurate than working on the assumption that the modulus of rigidity is absolutely the same for each propeller shaft along its whole length. It has been shown ("Notes on the Measurement of Shaft Horsepower," Institution of Naval Architects, March 18, 1910) that when the modulus of rigidity was taken at 12,000,000 lb. per square inch, the stiffness of the shaft so calculated was nearly always correct to within about 4 per cent. It is also important that the torsion meter should be permanently used on that length of shaft on which the twisting experiment was made when loaded by means of levers and weights, since the modulus of rigidity may vary slightly from point to point along the length of the shaft. It appears that experiments have yet to be made to investigate the difficult point, namely, whether the twist of a shaft is the same without and with the added pressure along its axis due to thrust. It must be remembered that the

conditions under which the shaft is calibrated by means of levers and weights are not exactly the same as those under which it normally works. When the shaft is calibrated in the works it is subjected to no end thrust such as exists when driving a propeller. The question then arises, Is the twist the same under the changed condition? Prof. Hopkinson writes: "It is a question of some importance whether the presence of this thrust affects the relation between torque and twist." The problem requires experimental investigation; on full-sized shafts the quality sought for would be small, but well worth the trouble of finding.

[THE TORSION DYNAMOMETER OF DR. ALFRED AMSLER.]

[This instrument was shown at the meeting of the Institution of Mechanical Engineers at Zürich on July 25, 1911, and it is described and illustrated in the Proceedings of the Institution of that date. I am enabled to reproduce one of the figures which sufficiently illustrates the construction, a courtesy on the part of the Institution for which I thank them. Two torsion dynamometers were shown, one with a capacity of 4,340 inch-pounds, and the other with a capacity of 6,944 inch-pounds, or 50 and 80 metre-kilograms respectively.

Fig. 100 is a longitudinal section of the smaller of the two. The torque is transmitted from the coupling F to the coupling H by means of the torsion shaft G. The two discs O and N, carried by the sleeve A, rotate with the coupling F, while the transparent circular celluloid scale U, attached to the disc M, moves with the coupling H. P and T are slits in the discs O and N through which the observer can see the scale, most conveniently by reflexion from a mirror as indicated, the scale being illuminated by a lamp. When the machine is running fast enough for the persistence of vision to be effective, the eye is able to see the scale apparently stationary, the slit T defining the particular division, which should be read without parallactic disturbance, while a widening of the slit T at one end into a window makes it possible to see the adjacent divisions and the number of degrees indicated. By carrying the eye round the circle the reading of the torsion can be made at any

desired part of the revolution, so that if the torque is uniform the same reading will be obtained in all positions; but if it is subject, as in an engine driven by one or two cranks, to cyclic variation this also may be detected and measured. Dr. Amsler gives the following figures for the smaller machine. The torsion shaft is made of a special spring steel of very high yield-point, that is, above 6,000 kilograms per square centimetre (90,000

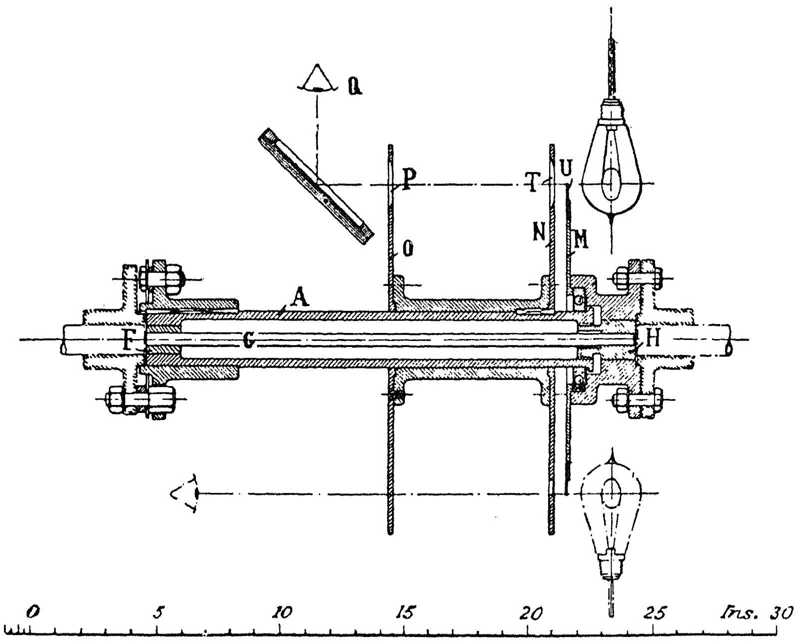


FIG. 100.

lb. per square inch). The length is 40 centimetres ($15\frac{3}{4}$ inches), its cross section is 12×12 millimetres ($\cdot 4725 \times \cdot 4725$ inch). With a twisting moment of 20 metre-kilograms (1,736 inch-pounds) the torsion is 20 degrees and the stress 5,200 kilograms per square centimetre (78,000 lb. per square inch), which is well below the yield-point.

Mr. H. H. Broughton, who spoke in the discussion, has given in the *Electrician* of December 12, 1913, an account of some torsion dynamometers that he has set up in Brighton. He used both the double-contact system and the illumination of

the scale by spark, but in his experience the simple optical arrangement of Dr. Amsler is greatly to be preferred.]

[THE TORSION METERS OF LUX, JOHNSON, AND THURSTON.]

[In *Engineering*, November 24, 1911, p. 715, there is an account of a development of the torsion dynamometer depending on the torsion of the shaft by Fritz Lux, the object of which is to make a horse-power meter which will integrate the power transmitted by the shaft to the propeller of a steamship. The torsion is measured by an electrical arrangement, and an electrically-operated counter is so contrived that the torsion angle and the angular speed are both factors which determine the speed of rotation on the indicating dial. Thus the record is one of horse-power hours. One of these was being fitted to the cruiser *Ersatz Kondor*.]

[On p. 605 of the same volume there is an account also of an electrically-worked torsion power indicator by Mr. C. H. Johnson, assistant works manager of Kelvin and James White. In this the angle of torsion is measured by the electrical effect of a sliding contact on a short piece of hard high-resistance wire. The contact is at the middle point when there is no torsion, and it moves one way or the other according to the direction of the torque. This produces a potentiometer effect which may be indicated as torque in any part of the ship. In the case of a ship with three propeller shafts the connections from the three shafts are brought to a single instrument board, where the work being done by each shaft may be determined very quickly.]

[In *Engineering*, November 8, 1912, p. 627, there is an illustrated account of the whirling table for aeroplane and propeller tests by Mr. A. P. Thurston, and made for the East London Technical College. In this the torque is measured by electrical means.]

CHAPTER XIII

THE CRADLE DYNAMOMETER

THIS form of work-measuring device was employed by the author in the year 1881. It was described in the journal of the Bristol Naturalist Society, 1883 (" Ergometer for Small

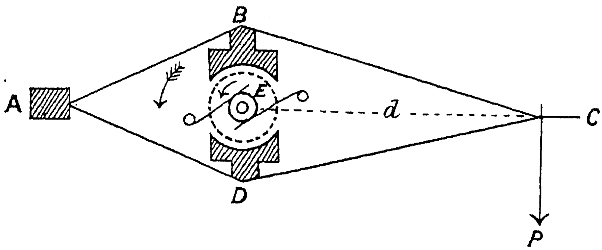


FIG. 101.

Electromotors"); a description will also be found in " Dynamo-Electric Machinery," by Prof. S. P. Thompson, 1884, p. 384. This type of dynamometer has been called the " Cradle Dynamometer," from the fact that in some cases the motor, when tested, is supported on a cradle free to oscillate. Referring to the diagram Fig. 101, the dynamo or motor BD is pivoted concentrically and balanced about the axis of the armature E. If this is a motor and the armature is driven in the direction of the curved arrow there will be a reaction as shown by the arrow P, and the force P $\times$ the distance d is a torque equal and opposite to that experienced by the armature. If it is a dynamo the force at P will be in the opposite direction, but the arrow shows the direction in which

a force must be applied to resist the torque felt by the field magnets. The invention was the outcome of experiments made by me, with a view to make a motor, or a dynamo, regulate the

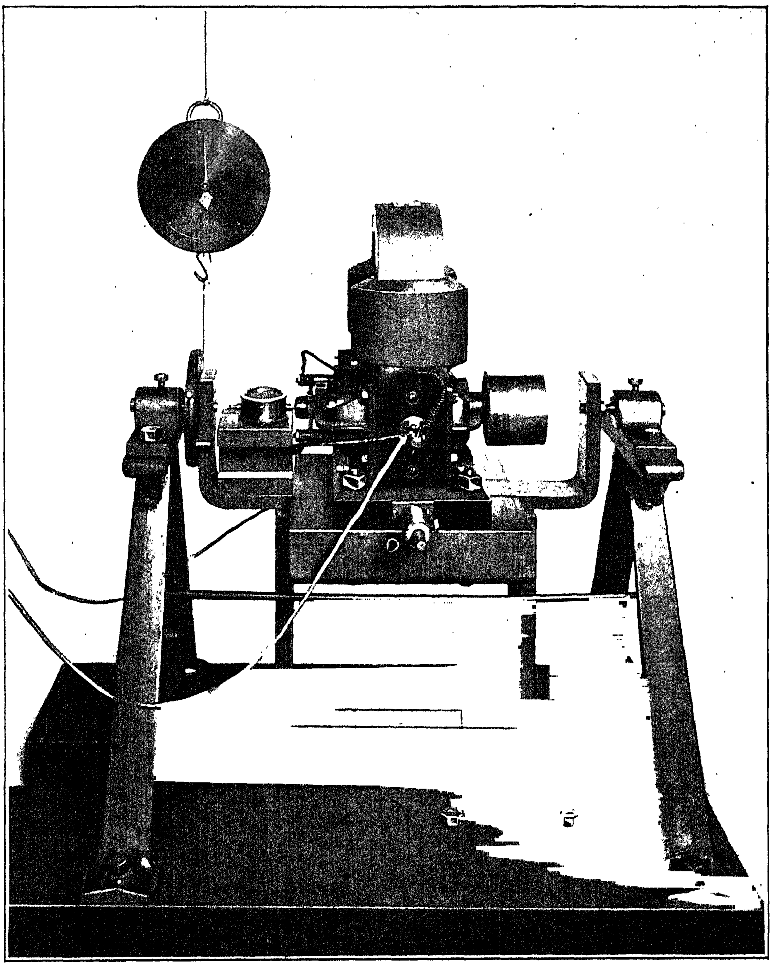


FIG. 102.

current supplied to it or delivered by it. The principle on which the machine works is as follows. The bearings on which the armature runs were supported on antifriction wheels so that the field magnet, which was carefully balanced, was free

to rotate, the ratio of the diameter of the bearings to the diameter of the wheels being about 1 to 7. The axle of the armature was attached, in certain experiments, to a small propeller shaft, the behaviour of which was to be tested under different conditions. When the motor drove the propeller, reaction was set up between the armature and the field magnet, and the latter was displaced through an angle the magnitude of which was proportional to the extension of a spring. The spring in some cases took the form of a torsional spring, the axis of which lay in the axis of the armature shaft, or of a spring balance, as in Fig. 102; also, in some cases, a weight fixed upon an arm projecting from the field magnet acted against the couple due to the field magnet and the armature. The arm was so placed that as the angle increased so too did the moment of the weight about the axis. When the number of revolutions per minute N were known and the effective radius in feet R at which the force in pounds F acted, the power was at once given by the equation

$$\text{Horse-power} = \frac{2\pi \text{RFN}}{33{,}000},$$

where $\pi = 3{\cdot}14159$ and 33,000 is the constant of Watt.

In the *Electrical World*, New York, of January 5, 1884, a dynamometer working on somewhat the same principle invented by Prof. C. F. Brackett, of Prinstown, New Jersey, is described. In this machine the cradle holding the motor was constructed of steel, and carried on knife edges. This is under certain conditions a better way of carrying a considerable weight than my own, namely, on antifriction roller bearings.

Dr. Drysdale has devised a direct-acting dynamometer of this type, described in *Engineering* of November 24, 1905. The knife-edge method of carrying a load employed does not appear to be as popular as it might be; some people have misgivings as to this method of support. We have but to consider the excellent results obtained in huge testing machines, in which the loaded beam is carried on steel knife edges, to be much encouraged to use the method in many other mechanical combinations. Of course, when the motor or dynamo is carried on knife edges, the belt drive must be in a vertical or nearly vertical line. When antifriction wheels are used, three on each side, the belt may be driven from any direction.

In the experimental plant designed by Dr. Drysdale * the principle of the balanced field magnet type of dynamometer (see p. 225) is employed. The generator, a four-poled eight-kilowatt direct-current machine by Westinghouse, designed to give 100 volts between the speeds of 750 and 1,600 revolutions per minute, was carried outside its bearings by ball races containing balls $\frac{1}{2}$ in. diameter. The field magnet was thus free to rotate. Two gun-metal arms fitted to the sides of the motor were provided with knife edges from which weights were suspended. The arms were fitted with two knife edges, one 2·625 feet from the axis, the other at 71·4 centimetres from the axis. When the machine made 1,000 revolutions per minute 2 pounds on the external knife edge represented 1 horse-power and 3 pounds on the inner knife edge represented one kilowatt. Knife edges on both sides enabled powers, whether positive or negative, to be measured with either direction of rotation of the armature. The electrical connections were flexible. Since this machine is fixed, the machine to be tested is carried on an adjustable slotted table which can be regulated for height by means of screw gear. The method of connecting the two machines is as follows. An American self-centring chuck forms a part of the connecting shaft. In order that the alignment shall be as perfect as possible, the end of the telescopic shaft is furnished with a small cone at its centre, which fits into the countersink of the shaft of the machine to be tested. The sleeve of the telescopic shaft passed through a ring supported from the floor, a space of $\frac{1}{32}$ in. being left round the shaft when it is correctly centred. The clearance when the little cone of the shaft was brought up to the counter-sink of the shaft showed at once whether one end of the shaft of the machine was on the axis. By the application of an ingenious optical method the other end of the shaft was brought into correct position, and by means of a weight movable on a rod projecting from the top of the field magnets the centre of gravity of the machine was adjusted.

When the machine was carried only on ball-bearings it was not sufficiently sensitive, and Dr. Drysdale found that tests

* See *Electrician*, p. 517, Vol. LXV., July 8, 1910; Drysdale, *Engineering* November 24, 1905.

could not be accurately made much within 5 per cent. In order that the sensitiveness should be increased, the machine was swung on knife edges. Finally, the machine was carried by means of a scale-beam placed above it supporting its weight through links on each side of it, each joint being of the knife-edge and flat kind. By means of the beam and links the weight of the machine was taken off the ball-bearings. The fulcrum of the beam was adjusted for height by means of a vertical screw and wheel. In my own form of balance dynamometer the antifriction wheels were large in comparison with the cylindrical bearings of the armature, in some cases as much as 10 to 1. For motors which were not very heavy the method of support gave good results.

Marcel Deprez suspended the field magnets of his machines of the Gramme type by knife edges on planes outside the bearings of the armature which was carried on separate bearings on each side of the machine. I have been recently informed by Messrs. Joshua Buckton & Co., the well-known makers of testing machines, that properly-constructed knife edges and planes will carry satisfactorily a load of five tons per inch run, the condition for getting a good result being that the highest quality of steel be employed, properly hardened, and ground dead true so as to ensure a bearing along their whole length.

Mr. J. Davis and Mr. F. Shaw have employed a cradle dynamometer designed by Mr. A. E. Moore, of the Manchester School of Technology, in a research on the output of a generator and the efficiency.* Also on the curves connecting iron loss torque and speed. This machine is carried on ball-bearings instead of knife edges. The results for sensitiveness appear to be excellent.

[A cradle dynamometer is described in the next chapter under "Motor-car Engine Tests," by Dr. Watson, F.R.S.]

* The Institution of Electrical Engineers, Manchester, Students' Section, April 19, 1910.

CHAPTER XIV

THE DYNAMOMETRIC TESTS OF MOTOR-CAR ENGINES AND HIGH-SPEED INTERNAL-COMBUSTION ENGINES

WHEN an internal combustion engine is tested for brake horse-power with the usual forms of rope brake, considerable vibration is set up and the readings of the balance scale are difficult to determine with accuracy. This arises from the fact that the rate of rotation changes during each revolution. It has been found that the vibration can be eliminated considerably, when testing the internal - combustion motor, by connecting it with some form of hydraulic brake, such as that of Brotherhood. The average torque per revolution is then given and a trustworthy result obtained.

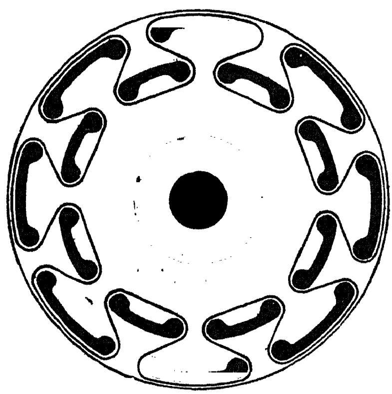


FIG. 103.

I find when making such a test that it is best to connect the motor to the dynamometer through two flexible couplings, such as those made under the Zodel-Voith patents (Fig. 103). In this coupling each of the two opposed flanges, one driving and the other driven, are furnished with eight curved projections having rounded ends; a continuous belt led to and fro between these projections forms the flexible connecting link. Another very early flexible coupling

consists of a cup-shaped flange connected to a flat flange by means of a disc of leather. Another is that used by Messrs. Crossley & Co. in connecting their gas engines with a dynamo. This consists of two opposed flanges furnished with studs ; the flexible link consists of a continuous rope, which is led to and fro between the studs alternately. Another simple and effective coupling is that of Siemens. The drive is made through a spiral spring wound on a shaft, connecting the motor and the dynamo. In the Proceedings of the Royal Society of Arts, 1910, p. 962, will be found the second Cantor lecture on "The Petrol Engine," by Prof. W. Watson, F.R.S. The indicated horse-power is found by means of the lecturer's excellent device, namely, a corrugated steel diaphragm, cooled by water circulating round the chamber in which it is fixed. This diaphragm takes the place of the piston of the ordinary steam engine indicator. The latter instrument is not suitable for very high speed engines owing to the inertia of its piston. Prof. Watson's diaphragm accurately shows the pressures in the cylinder when the whole stroke of the piston of the engine takes $\frac{1}{40}$ second and the pressure rises to 250 lb. per square inch in 0·003 second. By means of two mirrors, one rocked to and fro by the diaphragm and another rocked at right angles to it by a lever driven from the shaft of the engine, a beam of light is given two motions, which when projected on to a screen draws an indicator diagram, from which the indicated horse-power can be deduced. This is photographically recorded. The most valuable property of the diagram so found is that it shows the relationship of spark ignition to pressure developed. But it is with the dynamometric test of the petrol engine that we are concerned.

With some engineers the method of determining the horse-power by coupling the petrol motor with a calibrated dynamo has commended itself. This really means that the dynamo so employed gives an electrical output which, when certain corrections for losses have been made, exactly represents the horse-power employed to drive it. Probably with dynamos of considerable size, such as over 50 h.p., no great error would be introduced, but with smaller dynamos this is not the case. I think that Dr. C. V. Drysdale puts the matter clearly in the following paragraph, taken from a paper on "The Testing of

Electric Generators and Motors," *Engineering*, November 24, 1905 :—

"For small machines we are therefore thrown back on the measurement of the input and output. In the case of motors the mechanical output may be measured either by a transmission dynamometer or brake, while for generators the transmission dynamometer is alone available. There is, of course, the option of a 'calibrated' generator or motor, but this is pure begging of the question, as the calibration of this machine must be carried out by a dynamometer or brake, and cannot be regarded as constant."

I have found that the following method yields good results. Let the engine to be tested be coupled to a dynamo accurately balanced about the axis of its armature shaft, carried on anti-friction wheels, or, as Dr. Drysdale has done, on the arms of a balance furnished with knife edges. Then the dynamo is employed really as a brake, the current being so employed as to set up the required torque between the field magnets and the armature. Then, if the torque be known and also the revolutions per minute, the brake horse-power is known. I do not describe the above as by any means a new method. It was employed by me many years ago in the Millard Engineering Laboratory, Oxford. I think that Dr. Drysdale's method (which has been described on p. 221) of suspending the electric machine is excellent and very sensitive. One of the great advantages of this method of testing a prime-mover, in which the speed varies slightly in the course of each rotation, is that it may be continued over a long time, during which changes in advance, etc., of spark and vapour mixture can be made with ease. The proportion of the size of the dynamo to the engine of course must be considered, so that the dynamo may be well able to take the work put into it. The following will be found to be a convenient form of adjustable base for carrying many kinds of engines and motors. A base table furnished with T slots to which the engine may be secured by bolts is capable of five motions, three being motions of translation at right angles to one another and two of rotation, one about a vertical axis, the other about an axis at right angles to the line of the axis of the engine shaft, the slides in each case being of geometrical construction. A modern milling

machine has these motions, and the axis of the piece to be milled can readily be placed where desired. The design of this table was partly due to the late Mr. F. M. Newton, of Taunton. In order to make improvements in petrol engines, in which there are so many variables, the most careful dynamometric tests should be made in conjunction with the indicator diagrams, and expenditure on a complete testing plant should not be grudged by those who still desire to make further progress in the development of this class of prime mover.

[The internal-combustion engine, and especially the petrol engine, as used in a motor car or flying machine, when tested with a dynamometer depending on solid friction such as the Prony brake, is subject to special difficulty depending upon instability of speed. This type of engine for a considerable range of speed, including its useful range, develops an amount of work per revolution which varies but little with the speed. At extreme low speeds this may fall off in consequence of cooling of the hot gases and also if there is leakage, while at extreme high speeds it may fall off in consequence of insufficient supply of mixture and delay in the development of pressure with respect to the position of the piston. It results, therefore, that for a considerable range the horse-power is nearly proportional to the speed. In the same way the resistance due to solid friction is much the same over a very great range of speed, and so the horse-power absorbed will be for any setting of the brake very nearly proportional to the speed. If, therefore, an engine of this type is tested with a solid friction brake and a fair balance between the power and the resistance is obtained for some speed, this same balance very nearly will obtain for a large range of speed and the engine speed will be difficult to maintain. The engine may even run away or stop with small variations of power. For this reason it is much more convenient to test this type of engine with a dynamometer where the resistance is proportional to some higher power of the speed than the first, such as that given by fluid brakes, whether the fluid be air or water. If the power absorbed follows a cubic law, as with the fluid brakes, then a very small increase of speed will produce so great an increase of resistance that any slight access of power has an insignificant effect on the speed, which therefore is stable.

It has seemed to the writer that the Foucault current type of brake, described in Chapter VII., in which the increase of resistance with speed is less than it is in fluid friction brakes, could have the index of its law increased by 2, or nearly so, by including a dynamo in the brake the current from which would excite the electromagnets which are used for inducing eddy currents. If the magnetising coils were so proportioned that these only approached saturation at high speed, then the increase of resistance with speed would follow a higher law than it does with constant excitation and the stability of speed would be improved.]

[In the Proceedings of the Institution of Automobile Engineers of November, 1912, there is an account by Dr. W. Watson, F.R.S., of a carefully-conducted test of a petrol engine in which the load was taken up electrically, but instead of depending on the rating of the dynamo, a method which was found to be troublesome and unsatisfactory, the cradle system was adopted. The engine was connected by means of a Zodel-Voith flexible coupling to a nine-kilowatt electric motor with separately excited field. When this was used as a dynamo the load was absorbed in a large air-cooled resistance with step switches, so that the load, and therefore the speed, could be adjusted. It could also be used as a motor when the power needed to overcome the friction of the engine was being determined. As shown in Figs. 104, 105, the motor was supported by two turned cast-iron rings A resting on ball-bearing friction pulleys BB made of hard steel. As the cast-iron rings were not hard enough bands of spring steel were clipped round them, and these could be shifted in position if the bearing parts showed any wear after long-continued use. A transverse beam with a scale-pan D at each end made it possible to apply any torque to the motor magnets in either direction, and an oil dash-pot E was used to damp out vibrations. A large part of the weight of the motor was taken by a knife edge in the eyebolt at the top. The knife edge was carried by a wire attached to one end of a pivoted beam, the other end of which carried a counter-weight. With this compound support the motor is carried in a state of stable, not of neutral, equilibrium. This was convenient, for it was only necessary to use multiples of the pound on the scale-pans, while fractions were read by pointers moving over the scales F. A

deflection of 3·6 millimetres from the mean position corresponded to a torque of 1 pound-foot. The dynamometer worked so steadily that the pointer could be read with an accuracy of one or two tenths of a millimetre. I have to thank the author and

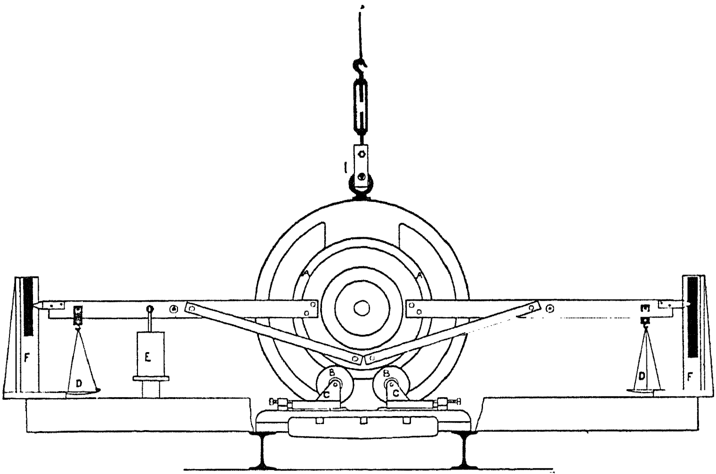


FIG. 104.

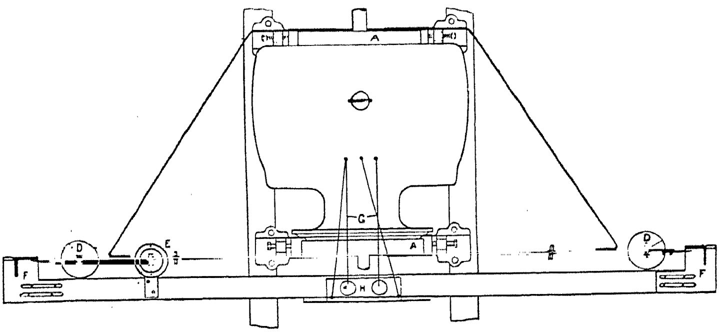


FIG. 105.

the Institution of Automobile Engineers for permission to use the illustration.]

[In the National Physical Laboratory Report for 1910, there is an account of a motor-testing plant in which the absorption dynamometer consists of a specially-constructed fifty-kilowatt direct-current generator mounted in the manner of the cradle

dynamometer on knife edges, so that the power absorbed may be determined without depending on the calibration of the dynamo. A water-cooled friction brake is also provided.]

THE WIMPERIS ACCELEROMETER.

[In recent years the development of the accelerometer has led to a new and extremely practical and quick method of testing the brake horse-power of motor-car engines in position in the car and on the road.

Lanchester's accelerometer is described in the Proceedings of the Institution of Automobile Engineers, 1909—10, Vol. IV., while that of Wimperis will be found in the Proceedings of the same Institution for the year 1914.* As the Wimperis accelerometer is the one best suited for this kind of test, I shall describe the construction of this one by the aid of Figs. 106—108, for which I am indebted to the manufacturers, Messrs. Elliot Brothers.

A and B are two discs geared together and delicately pivoted like wheels in a watch. The circle in each represents a hole which puts them out of balance. It will be seen that these holes are symmetrically placed with respect to the horizontal line, and, owing to the gearing, they must always be so. Now suppose the instrument in which the discs are pivoted to be given an acceleration in the direction of the arrow P, then, owing to the want of balance of the discs, the heavy side of each will have an inclination to remain behind or the holes will tend to move ahead. They will tend to turn in opposite directions, and the gearing agrees with this. If one is controlled by a hair-spring like the balance-wheel of a watch, it will turn so far that the combined torque due to the two unbalanced discs will be equal and opposite to that exerted by the spring. If, however, any acceleration is applied in the direction of the arrow Q, the tendency of the two discs will be equal and in the same direction of rotation, but as this is inconsistent with the gearing this acceleration has no ultimate effect. Similarly, acceleration perpendicular to the plane of the paper can have no effect either.

* See also British Association Report, 1910; Proceedings of the Institution of Civil Engineers, Vol. CLXXXVIII., and "The Principles of the Application of Power to Road Transport," by H. E. Wimperis. London. Constable and Co., Ltd., 1913

It will be clear, then, that such a construction picks out acceleration in the direction of the arrow P, or retardation which is negative acceleration, and responds to it, but

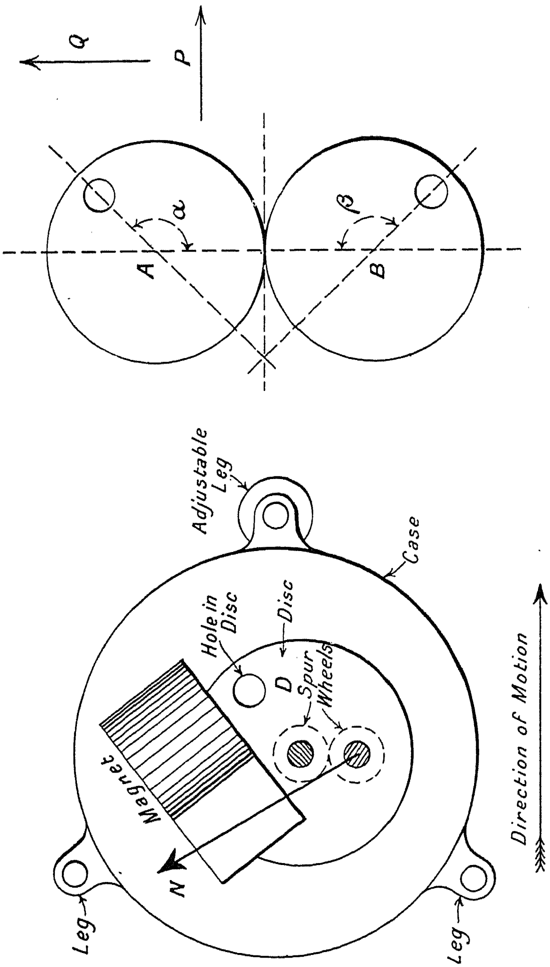


FIG. 106.

is insensible to acceleration in any direction perpendicular to this. The left hand side of Fig. 106 is a plan diagram of the construction of the instrument. Here the two spur-wheels are seen geared together. The upper one carries a larger copper disc in which the unbalancing

hole is made. The lower one carries a hand which moves over the dial seen in Fig. 107. The positive weight of the hand is dynamically equivalent to the negative weight of a hole on the other side of the centre. The hair-spring is so attached as to make the pointer lie over the zero of the acceleration scale when the instrument is level and at rest and then the angles α, β are then each of them 180 degrees. With acceleration the pointer moves to the left, while with retardation it moves to the right. As the reaction of the spring is propor-

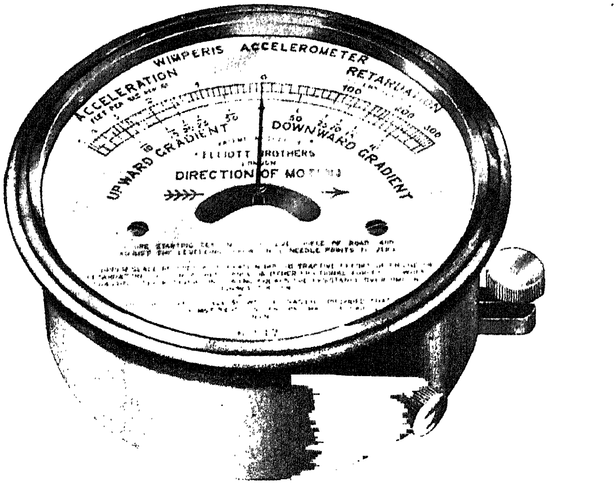


FIG. 107.

tional to the angle of deflection, while the arm at which the centres of gravity of the moving elements acts varies as the cosine of this angle, the scale of acceleration and retardation is not one of equal parts, but one based on the cosine law. The copper disc moves between the poles of the magnet as shown, thus experiencing a force of retardation proportional to its velocity, and so vibrations are damped out and steady readings obtained. The instrument stands upon three feet, the one in the direction of travel being furnished with a levelling screw to bring the pointer to zero when all is quiet.

Acceleration is literally the rate at which velocity changes, and it is measured as velocity per second or feet per second

per second added or subtracted. As the action of force upon mass is to produce acceleration, the term "acceleration" is often used, conveniently but wrongly, as though it were force. In dynamical units the acceleration due to gravity is 32·2 feet per second per second, while the acceleration due to diluted gravity down a frictionless incline of 1 in 10 is 3·22 feet per second per second.

It is often convenient where acceleration is measured with a view to determine forces to reckon the force as so many pounds

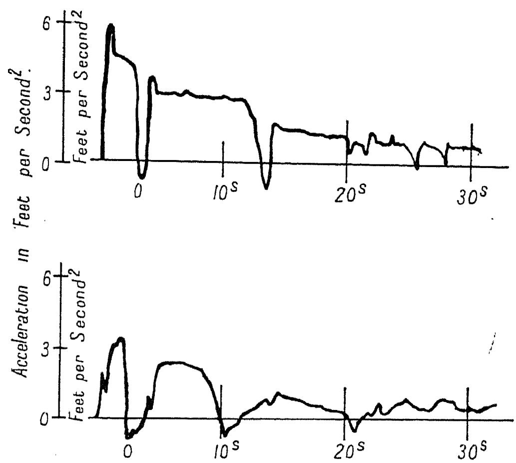


FIG. 108.

per ton of the weight of the car being examined. Thus, if the acceleration of the car is 1 foot per second per second, this acceleration would be caused by $\frac{1}{32 \cdot 2}$ of its weight, or by a force measured in gravitational units as $\frac{2,240}{32 \cdot 2} = 69 \cdot 6$ pounds per ton in excess of that needed to overcome resistance to steady motion. The number 70 is so near 69·6 that it is commonly taken as the number of pounds per ton corresponding to unit acceleration—or retardation if the action of a brake is being considered.

If an accelerometer is placed upon the floor of a car and care-

fully levelled when the passengers are in their places, and then the car is started on a level road in the usual way, the needle of the instrument will show the acceleration through all the operations of letting in the clutch, and during the steady increase of speed on each gear, dropping for a moment to a negative quantity as the gears are being changed. Two examples taken with a recording accelerometer are shown in Fig. 108. These were taken on a three-speed touring car at Brooklands. The upper curve shows the acceleration when speed was got up as quickly as possible, while when the lower one was obtained this process was not unduly hurried. The greatest acceleration is due to the coming into action of the clutch, and the next is got with the low gear. It is the drop of acceleration to a negative quantity with which we are immediately concerned.

Taking a car first on a level road and driven at any desired speed, the speed must first be read on a speedometer, and then when all is going steadily the clutch is suddenly thrown out of action and the accelerometer read. The car meets with resistance due to road friction, to internal friction in the mechanism between the road wheels and the clutch, and to wind or air resistance, and this last at high speeds is the most important. In virtue of these resistances its acceleration, which was zero when it was moving at a uniform speed, becomes suddenly negative, and this may be read as so many pounds per ton. Up to the moment of declutching, when the velocity was uniform and the acceleration zero, the engine was providing the power to overcome these resistances, *i.e.*, it was applying a force of the ascertained number of pounds at the speed of the car. The following example is taken from the makers' pamphlet :—

Weight = 1·67 tons

Tractive resistance by accelerometer = 165 pounds per ton = 275·5 pounds.

Speed = 44·78 miles per hour = 65·7 feet per second

Brake horse-power $= \dfrac{275{\cdot}5 \times 65{\cdot}7}{550} = 32{\cdot}9$, or 33 as nearly as it can be ascertained ;

or, more shortly, $\dfrac{W \times R \times V}{375}$ = brake horse-power,

where W = weight in tons
R = tractive resistance in pounds per ton
and V = velocity in miles per hour.

The following values have been found for R :—

Clean wood or hard macadam . .	70 lb. per ton.
Muddy and sticky road . . .	95 ,, ,,
Road metal partly rolled . .	120 ,, ,,
Loose road metal not rolled . .	200 ,, ,,

So far we have been considering a level road only. If the road is not level, the real acceleration of the car in feet per second per second will be altogether different; nevertheless the readings of the accelerometer as indicating forces applied by engine or by brake will be true, and for this purpose the slope of the road has no effect. This is a surprising result to many, but a moment's consideration will show that, in so far as the actual movements of the car are affected by the slope tending to move the accelerometer needle in one direction, the consequent slope of the instrument will act to an equal extent in the opposite direction. That this must be so is most readily proved by considering a model cart moving without friction down an inclined plane. If such a cart were to carry a pendulum, the pendulum would lean forwards relatively to the cart if this were at rest, but if it were liberated and were free to accelerate under gravity, meeting with no friction on the plane, the reaction between the plane and the cart and all its contents would be perpendicular to the plane and the pendulum would hang square to the cart, the same as if they were on the level and at rest. The pendulum is an accelerometer of simple construction, and so the reading on an incline with no forces acting except gravity is zero. As the accelerometer acts dynamically as a pendulum in relation to fore and aft motion and acceleration, it also is its equivalent as an indicator of slope. The scale of the instrument is therefore provided with a lower set of divisions which will indicate by the position of the pointer the gradient on which the car is standing or on which it is moving, provided that the speed is uniform or the acceleration zero. It should be pointed out that there is one error in the statement as to horse-power as made. The weight of the car in tons should have a very small amount added to it corresponding with the rotational energy of the road wheels

and revolving mechanism up to the clutch. The tyres, for instance, should be counted nearly twice over, and the other parts in a proportion less than this. If W is taken to mean the weight in tons increased in this way by the right amount, then the expression given is correct.

It may be well to add the statement that in the instruments as at present made the large amount of dead and useless material of the unbalanced discs is no longer employed, but instead the same departure from balance is obtained by the use of a short light arm with a small weight at the end.]

CHAPTER XV

SHIP MODEL DYNAMOMETER

THE very important researches of William Froude, made at Torquay, on the friction experienced by ships moving through water have borne an excellent harvest, and an experimental plant very similar to Froude's original one at Torquay was erected by the Admiralty at Haslar, where excellent and constant work is carried on by his son, R. E. Froude, F.R.S.

The importance of the work has been thoroughly recognised by foreign Powers and private shipbuilding firms. In order that the behaviour of a model of a ship may be compared with the behaviour of the ship itself, Froude employed a "scale of comparison," as he called it; this was based on the stream line theory, and is thus stated :—

"If the ship be D times the dimension of the model, and if at speeds V_1, V_2, V_3 the measured resistances of the model are R_1, R_2, R_3, then for speeds $V_1\sqrt{D}$, $V_2\sqrt{D}$, $V_3\sqrt{D}$ of the ship, the resistance will be D^3R_1, D^3R_2, D^3R_3. . . ." Froude applied the expression "corresponding speeds" to the speeds of the model and ship. An example taken from Froude's paper in Vol. XV. of the Transactions of the Institution of Naval Architects, and quoted by Sir William White, "Manual of Naval Architecture," 1894, p. 478, will make this clear.

In Fig. 109 is shown a "curve of resistance," in which abscissæ represent speed in feet per minute, set off along XY, while the ordinates represent the resistance of a ship or a model of a ship in pounds at different speeds. By means of a traction dynamometer, which will be described later on, the resistance of a model, say, at 240 ft. per minute is found; this value is set

off as an ordinate, *ad*, at the point *d*, which corresponds to the speed. Several points, such as *a*, are found for different speeds, and through these the curve AA is drawn. From a curve so formed the resistance can be found for any speed included in the experiments. From the immersed surface of the model and an experimental determination of the coefficient of friction the frictional resistance for each speed can be calculated.

Next the frictional resistance is set off from the base line XY for each speed on the same scale. For a speed 240 ft. per minute *db* represents the frictional resistance. Thus a curve BB of frictional resistance is found for the model. From these

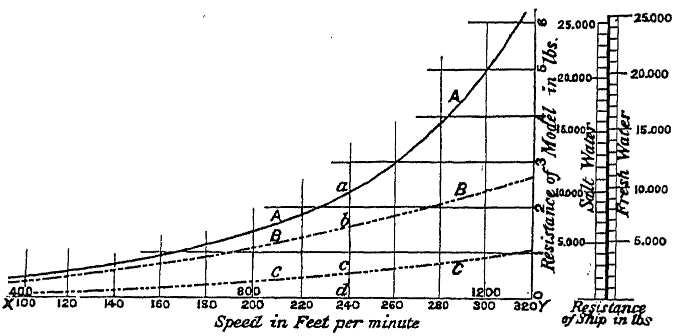


FIG. 109.

data the resistance of the full-sized ship can be found. Froude made his experiments on the *Greyhound*; the scale of the model was one-sixteenth that of the ship: so that for the scale of comparison $D = 16 : \sqrt{D} = 4$; and therefore the "corresponding speeds" of the ship will be four times those of the model. In the figure the speeds for the model are marked below XY; and the speeds of the ship above this line Then

$$\text{Resistance of ship} = (16)^3 \times \text{resistance of model.}$$
$$= 4{,}096 \times \quad ,, \quad ,,$$

This change, therefore, simply amounts to an alteration in the scale of measurement of the ordinates of the curve AA; and whatever length represents 1 pound for the model must represent 4,096 pounds for the ship. On the right of the curve diagram the correction is shown by the scale of "resistance of ship"—in

fresh and in salt water. This scale gives values for resistance in fresh water and sea water, the resistance in salt water exceeding that in fresh water in the ratio of the density of sea water to fresh water. The scale was employed since in the experimental tank fresh water was used. Since the length of the ship greatly exceeds that of the model a correction is required on this account. The frictional resistance of the ship is calculated for the various speeds, her actual coefficient of friction being made use of, and these are set off, on the proper scale and on ordinates representing the corresponding speeds, downwards from the curve BB, which represents the frictional resistance of the model; through the points then determined the curve CC is drawn. Then, to determine the resistance of the ship at any speed instead of measuring from the base-line XY, it is necessary to measure from the line CC. I am indebted to Sir William White, F.R.S., for his kind permission to reproduce the figures and description from his "Manual of Naval Architecture," p. 479.

The mechanical problem which William Froude set himself to solve was to make exact models of full-sized ships and then to tow them through water at a uniform known velocity. The force required to tow them was accurately recorded during the whole transit of the models. The exhaustive paper of William Froude, F.R.S., in Vol. XV. of the Transactions of the Institution of Naval Architects, and that of his son, R. E. Froude, F.R.S. (Institution of Mechanical Engineers, February 2, 1893), should be carefully read by any one wishing to appreciate the material outcome of genius and ability. Only enough will be stated here by way of an outline of this splendid piece of work.

The different parts of the model ship-testing apparatus are these:—The water way; the experimental carriage; the engine and hawling gear; the governor of the engine; the model-shaping machine; the copying apparatus; the forming of the model; the weighing of the model; the record of an experiment.

The water way is in the form of a canal. At the Admiralty Experimental Works, Haslar, it is about 400 feet long, of nearly uniform section, having vertical sides. On each side of this tank rails are laid down, and on these, which

are nearly 21 feet apart, the experimental carriage runs. In the first ship model apparatus the experimental carriage was carried from the roof built over the tank, which was constructed at Torquay by William Froude. The experimental carriage is equipped with recording dynamometric apparatus, so that the pull is recorded by means of a pen on a sheet of paper which covers a cylinder, driven from one of the flanged wheels of the carriage; on the same paper a simultaneous record is made by a pen actuated by an electromagnet, the circuit of which is broken by a clock at definite intervals; another pen records a broken line showing distances passed by the carriage. The ordinate of the diagram produced is directly proportional to the force of traction at any instant. The carriage is of peculiar construction; the members of the trusses of which it consists are formed out of wood, in the form of trunks or boxes about 4 inches square in cross-section, the sides being $\frac{3}{8}$ inch deal put together with shellac varnish and screwed. The strength of this structure for its weight is very great, and it has stood the test of many years of constant work. Sir William White when commenting on the construction said: "It was indeed an admirable example of structural arrangement with so light a material, which had not been supposed to lend itself to the girder method of construction."* The carriage is shown in Fig. 110.

The carriage is propelled by means of a wire rope, led over a grooved sheave, driven by a 10-inch Tower spherical engine. The range of speed for towing the carriage is between 100 and 500 feet per minute, and a speed as high as 1,200 feet per minute has been reached. The governor is quite different from all other centrifugal governors. The centrifugal force due to two masses of metal is opposed by a spiral spring; when the adjusted distance is reached by the masses they cause a disc, sliding on the spindle of the governor, to engage frictionally with a disc, to which a link is attached which actuates the supply valve of the engine. This is slightly rotated and the supply of steam lessened. The performance of this governor is excellent and reliable. A similar principle is embodied in the driving mechanism of one form of the phonograph. In

* Proceedings of the Institution of Mechanical Engineers, February 2, 1893, p. 43.

this case clock-work drives the phonograph by frictional engagement through a minute centrifugal governor only when the desired speed is not exceeded, and thus the recording cylinder is driven at a uniform speed.

The model ships are made of hard paraffin wax about 14 feet long, and when finished about 1 inch in thickness, an allowance of $\frac{1}{4}$ inch being made for finishing them accurately to shape, which is effected by a model-shaping machine (Figs. 111 and 112). Fig. 112, which is a transverse section, is on a larger scale. This machine consists of the model table, which has a maximum travel of 20 feet, on which the cast model is fixed keel upwards. On each side of the model two fly-cutters revolve at 2,700 revolutions per minute; while the unshaped cast model traverses between these cutters they are caused to follow the lines of the ship, drawn on paper, so that contour lines are cut on the model. After a sufficient number of these have been cut the model is removed, and the surface finished by tools of the spoke-shave type in such a way that all the contour lines cut by the fly-cutters are joined. The movements of the table, with the model on it, and the copying point are all under the control of the operator. The table is traversed by means of a piston moving in a long cylinder, the piston-rod being a continuous steel piano wire, led out through glands at each end of the cylinder and then over pulleys. The piston is moved by oil under a pressure of 14 pounds per square inch, pumped by a small vane pump. This machine should be seen to appreciate its numerous points of excellence. The models are cast in moulds made of white clay contained in a long rectangular box. The melted wax is used at a temperature of about 160° F. After the model ship is finished it is weighed, so that correct additional weights may be put into it to give it the right depth of flotation. In order that the behaviour of the model ship may give results from which the behaviour of the full-sized ship may be truly predicted, every measurement connected with the model ship must be as accurate as possible; this accuracy has been attained in a high degree in the testing plant at Haslar. Many other points are also investigated, such as screw efficiency and the effects due to wave-making.

Referring again to Figs. 111, 112, V is the traversing carriage with the paraffin model on it, being shaped by the

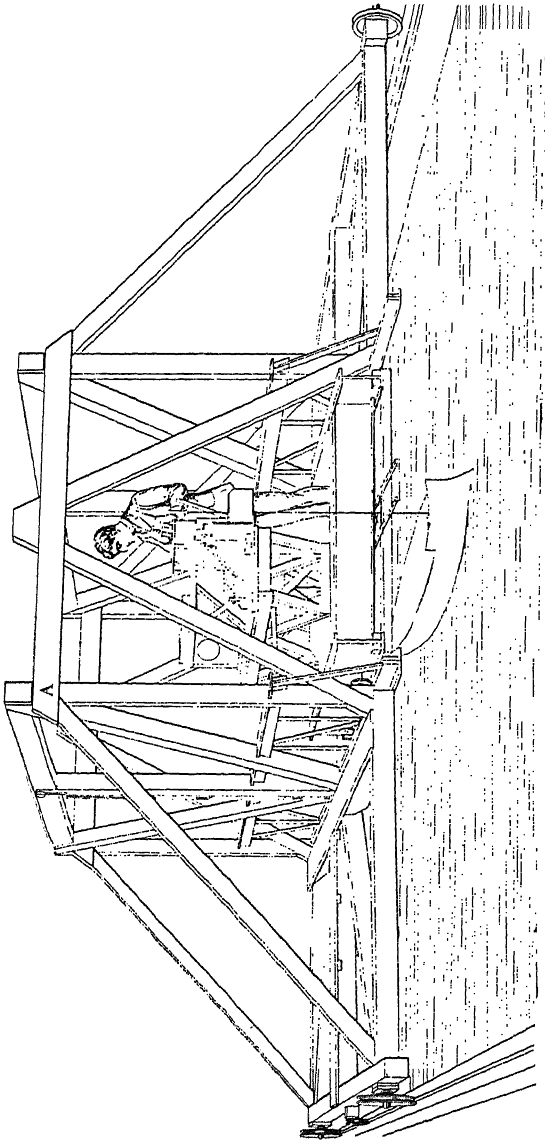


Fig. 110.

revolving cutters, carried on the spindles RR. AA are wheels for raising the cutters symmetrically. These are rotated by the gearing B and cross-shaft DD, on which slides

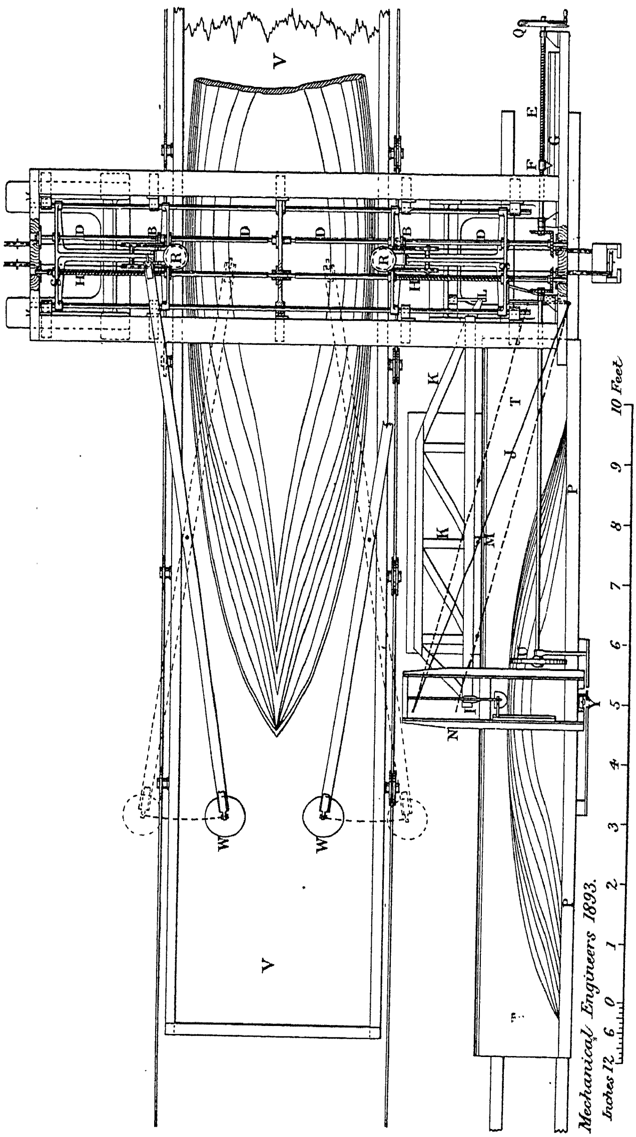


FIG. 111.

the spur wheel B, fitted with a sliding feather. The rotation of D is effected by the hand-wheel Q through mitre gear; the hand-wheel Q also moves the indicating nut F through the

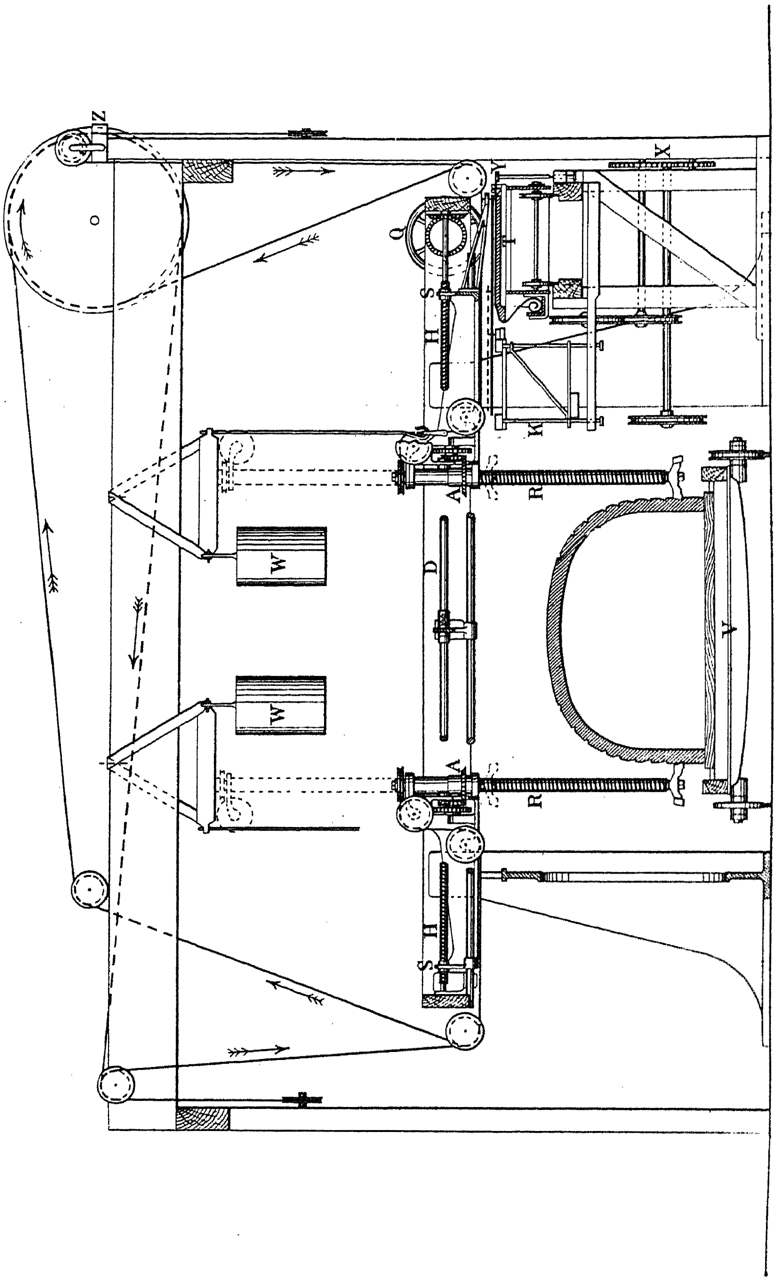


FIG. 112.

distance of the vertical rise or fall of the cutters, so that they can be set by this means at the levels of the successive water lines marked on G. The lateral motion of the cutters, which is symmetrical, is controlled by a right and left handed screw H, engaging in the nuts SS. This is worked by mitre gear through the hand-wheel U. The cutters always work from midships of the model towards its ends, so that they continuously move inwards; their sliding friction is reduced by the counter-balance levers and weights WW. The fulcrum M of the copying lever J (indicated by a single line) is mounted on the frame K, which is also a lever pivoted at L, just beneath the line of motion of the cutter end of the copying lever, being held at the other end at the point I, in the line of motion of the tracer, by the frame N. The position of N in the direction of the travel of the tracer is regulated by the roller Y, held in contact with the back edge of the batten P by means of a light weight over a pulley, the front edge of the batten being adjusted to a uniform distance from the centre line of the drawing. The proportional longitudinal travel of the drawing fixed to the table to that of the model is regulated by the change wheels X, the teeth of the cog-wheels being always kept in contact by a weight acting through a line over a pulley; thus any play of teeth is avoided. The motion of the table carrying the model is given to it by a piston and cylinder actuated by oil, pumped by a small centrifugal pump Z (this has been previously described). The tracer is not a point but a circle of the same diameter as the cutters, and its edge is kept in contact with the lines of the drawing while the cutting is made; if the plan and model differ in length, then in place of a circle an ellipse is used, the axes being proportional to the relative length of the plan and the model. No difficulty is experienced in keeping the edge of the circle or the ellipse in contact with the lines of the drawing, or in manipulating the hand-wheels and foot-gear, which are all in easy reach of the operator.

The example set by the British Admiralty has been followed by several foreign Governments—France, Italy, Germany, Russia, and Japan. Also some shipbuilding firms have added a complete model-testing plant to their works: the pioneers in this are Messrs. William Denny & Sons. Quite recently a testing tank for ship models has been built at the National

Physical Laboratory, Teddington, where every detail has been thought out with great care, much being due to Mr. Horace Darwin, F.R.S. It was initiated by the Institution of Naval Architects, Mr. A. F. Yarrow offering, subject to certain conditions, the sum of £20,000 to defray the cost of its construction and its outfit (Report of the Experimental Tank Committee (1908), Institution of Naval Architects).

Instead of the towing engine of Froude an electrically-driven motor is employed. This motor forms part of the experimental carriage, so that any vibration due to a long steel-rope drive is obviated. When we consider the enormous tonnage of the war and mercantile shipping belonging to England, the great importance of such a plant, where different naval architects can obtain trustworthy and careful tests of their designs, cannot be over-estimated.

Notes on Experimental Tanks.

Chronological order of eight Experimental Tanks :—

Torquay, first of such tanks	1872
Dumbarton, Messrs. Denny & Sons	1882
Admiralty Tank, Haslar	1886
Spezia tank	1889
Washington tank	1898
Bremerhaven tank	1900
Berlin tanks	1902
Uebigau tank	1904
National Physical Laboratory Tank in construction.	1910

[The National Physical Laboratory tank and equipment at Bushey have been completed since the author wrote his account of Froude's work. As the general design is based upon that of Froude, though in detail there are alterations and improvements, and electric motors in particular have facilitated the operations, it seems unnecessary to describe these in detail. It is sufficient to refer to the collected researches of the National Physical Laboratory, Vols. VI. to IX., or to the Transactions of the Institution of Naval Architects of 1910 and succeeding years.]

Details of Experimental Tanks (from the Institution of Naval Architects, 1908).

	Effective Length.	Depth of Water.	Breadth of Water.	Area of Cross-section.	Breadth of Building.	Breadth of Carriage.	Weight of Carriage.	Velocity.	Horse-power on Carriage.
	feet.	feet.	feet.	square yards.	feet.	feet.	tons.	feet per second.	
Berlin . . .	479	11·5	34·0	29·5	34·6	19·8	13·5	23	20
Uebigau . .	289	11·3	21·4	22·3	30·1	23	5·5	16·6	20
Paris . . .	528	14·0	32·8	36·3	41	34	25	15·5	100
Haslar . . .	400	9·0	20	20·3	28·9	22	1·25	16	18
Denny's . .	275	10	25·1	28	26	4	—	—	—
Clydebank . .	400	9·5	20	20	28	21	2	16·6	12
Bremerhaven . .	476	10·5	19·8	22	26·3	20·6	—	15·6	20
Japan . . .	450	12	20	26·6	—	—	—	20	30
Washington . .	384	14·75	44·6	46·5	47·6	45	32	30	200
Michigan . .	300	10	22	23·3	—	—	—	13·3	—
Suggested tank for the N.P.L.	500	12·5	30	40	42	31	10	25	50

The effective length means the length at the full depth, and does not include the docks or the sloping beach at the end.

CHAPTER XVI

THE AERONAUTIC DYNAMOMETER

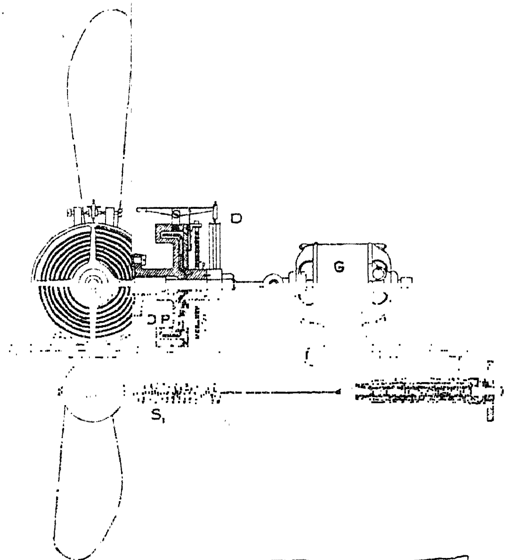
THROUGH the courtesy of the secretary of the Advisory Committee for Aeronautics, 1910, I am able to give a description, with figures, of the dynamometer used in experiments made on air propellers. Problems in air resistance have long ago been studied by employing some kind of whirling table, that is an arm, revolving in a horizontal plane, carrying the surface opposed to air friction and pressure at its end. This method was employed by Robins in 1746, Hutton in 1787, Smeaton in 1759, Hirn in 1854, also more recently by Dines in England and Langley in America. When an arm of great length is employed as the carrier of the surface or propeller, a close approximation to the conditions existing in an actual air machine can be obtained. The radius of the arm of the whirling table at the National Physical Laboratory is 30 feet; it rotates within a galvanised iron shed 80 feet by 80 feet, so as to be free from atmospheric disturbance.

The 30 foot arm is built up of light steel tubes tapering from 1½ inch diameter at the axis to 1 inch at the extremity; they are 12½ inches apart, and are connected together by struts (Figs. 113, 114 and 115). The central post rises 6 feet above the tubes, and the stiffening is effected by connecting the tubes to a cantilever built up of light angled irons furnished with cross-bars and steel-wire ties. A 14 horse-power electric motor drives the arm through worm-wheel reduction gear of 28 to 1.

In order to prevent the post being strained by the inertia of the arm when stopped, the post is divided above the worm-wheel, the upper and lower parts being connected by a ratchet gear which allows the full rotation of the arm when the motor

is stopped. The arm can be driven from five to thirty revolu tions per minute ; this corresponds to a speed of the propeller-testing mechanism of from ten to sixty miles per hour. Current is supplied to the testing mechanism through slip-rings fixed to the central post, and wires connecting them with the motor, etc. The propeller shaft is driven by a half horse-power motor, carried by the arm 8 feet from its extremity, the motion being given to the shaft by a belt. This motor is furnished with a speed-regulator worked from the observing table, so that when an experiment is being made both the speed of rotation of the propeller and its forward movement can be separately controlled.

The propeller dynamometer is shown in Figs. 116 and 117 ; it is so designed that the torque on the propeller shaft, due to a given thrust, is recorded on a drum. To effect this the ball-bearings carrying the propeller shaft are supported by a link motion which allows a small horizontal movement of the shaft, this motion being controlled by the spring S_1. The pulley P, by which the shaft is driven, is mounted on the bracket B, and transmits motion to the shaft through the outer casing of the oil dash-pot DP and the coiled spring S_2. On the face of the casing of the dash-pot a lever is carried ; this is furnished with a pencil, which can scribe a line on paper on the drum attached to the propeller shaft. The end of the propeller shaft is connected to the armature of a small generator G, so that the speed of rotation can be found from the readings of a voltmeter on the observing table. In making an experiment on a propeller the tension of the spring S_1 is set to a given value and the speed of the whirling kept constant. The speed of the propeller shaft is gradually increased by means of a regulator until the thrust of the propeller is sufficient to balance the pull of the spring. When this balance has been effected the propeller shaft moves back on the link motion through a small distance. This is recorded by a movement of the pencil parallel with the axis of the propeller shaft. The trace thus made on the drum, which shows the circumferential motion of the pencil, is read and the torque corresponding to a given thrust deduced. In order that the desired thrust may be known the lever of the link motion by means of contacts shows either a red or a green light on the whirling table.



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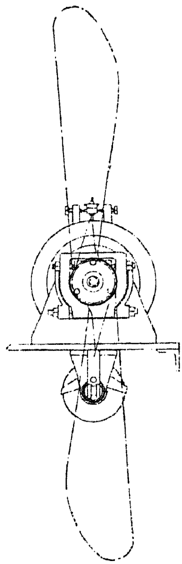

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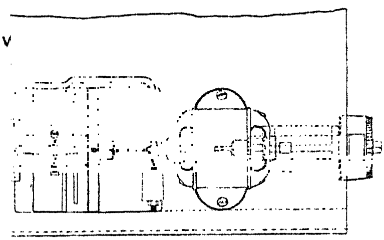

Air-propeller Testing Plant at Messrs. Vickers, Sons and Maxim's Works, Barrow in Furness.

This apparatus consists, as will be seen from Figs. 118 and 119, of a steel cantilever, accurately balanced and suspended in such a manner that it is free to revolve about the head of a cast-iron column. The point to which the suspension rods converge is a steel bracket to which is fastened a steel tube, constructed of rolled-steel plates, but jointed and riveted. At the head of this tube there is a ball-bearing which supports the entire weight of the moving portion of the structure, a guide from the bottom end being supplied by four horizontal rollers carried on cast-iron brackets bolted to the lower end of the steel tube and rolling on a turned belt on the column.

The arm which revolves is built up of steel angles, and is provided with a covered-in observation station at the centre. which contains the 100 horse-power motor and the recording instruments. At the extreme end of the arm, and 110 feet from the centre, there is a steel platform carrying the bracket and bevel gearing for driving the propeller, the power being transmitted along the arm by a line of steel shafting. The other end of the arm terminates in a sheet iron ballast tank at a radius of 56 feet, by means of which it is possible to balance the whole structure.

The revolutions of the propeller may be varied from 500 to over 1,000 revolutions per minute. The speed of the propeller through the air can be regulated by means of screens, so as to conform to the conditions for which it is designed, which speed may reach seventy miles an hour.

A system of accurately measuring the thrust of the propeller is included in the design of the bracket and gearing, the propeller shaft being allowed to move forward against a spring, which movement is mechanically recorded in the observation station. Experience shows that the thrust can be accurately measured to within 1 per cent. in a total thrust of 500 pounds.

The gear is made with a reverse so that the efficiency of the propellers can also be tested for going astern.

The whole of the conditions for this machine are exactly similar to those of a ship running in a straight line through the air, as an ingenious method for compensating for the circular

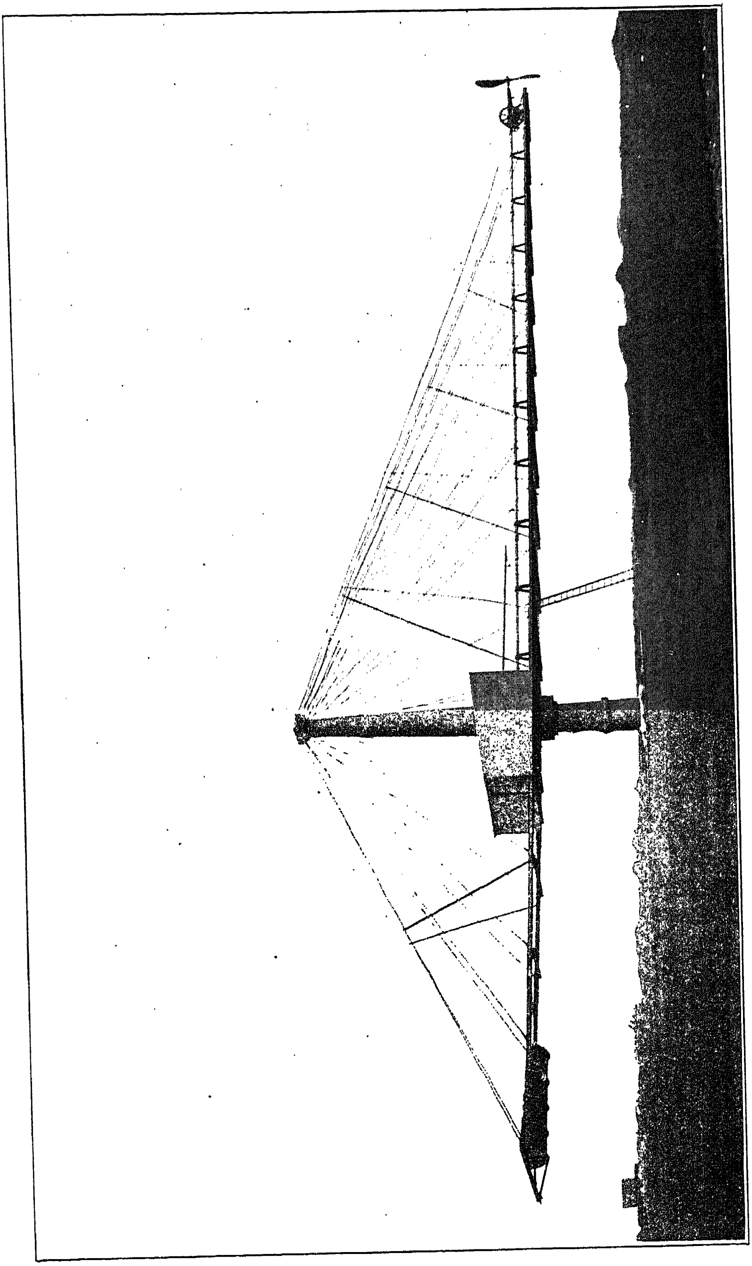

FIG. 118.

motion of the propeller has been arrived at, without which, in similar machines, that portion of the propeller blade nearest

FIG. 119.

the centre column is caused to travel less rapidly than the outer portion.

Provision has also been made for attaching a gondola to the

platform ahead of the propeller, so that the results obtained from the machine may be relied upon as being exactly similar to those which will actually prevail when the propeller is placed on the ship, astern of the gondola. By this means the exact condition of the propeller when it is thus fitted in position in the ship may be imitated.

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